Pareto Improving Tax Reform

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August 29, 2019

Abstract

For almost all competitive economies with externalities, there exists a Pareto improving anonymous linear tax policy. Taxes alter equilibrium prices and allocation; if marginal external effects are rich at equilibrium, the array of welfare changes induced by taxes are rich enough to include a Pareto improvement.

1 Introduction

Since Hayek (1945), prices are understood to transfer relevant localized information so as to coordinate economic activity. The result of Arrow (1951) and Debreu (1951) states that, under certain preconditions, prices do so efficiently. One such precondition is that markets are complete. Arrow (1969) defines an exchange economy to have complete markets if all goods and services pertinent to utility and production are marketable. This paper studies the efficiency of the price mechanism in an economy with consumption externalities: markets are incomplete because economic agents' actions affect others in a way that is not marketable. Markets are assumed to be competitive. Consumers are numerous enough that each of them consider the impact of their action on prices as negligible. Also, they do no internalize the impact of their actions on others and hence they do not behave strategically.

Geanakoplos and Polemarchakis (2008) proved that, for almost all economies with additively separable externalities, there exists an anonymous tax policy that makes everybody in the economy better off. This paper extends their result to a more general class of economies, namely economies with non-separable externalities. Importantly,

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non-separable external effects affect choices. As a result, there is hope for the identification of preferences from market data on consumption and prices. This, in turn may help the policy authority to acquire the relevant information for the design of Pareto improving tax policies. It is hoped that the result of this paper takes a step in this direction.

A different approach from the one used so far seemed inevitable. The crucial difference between the effects of additively separable (linear) externalities and non-separable externalities is that while the former affect the level of utility only, the latter may also affect choices. Non-additively separable externalities introduce intricate interdependencies, which nests the effect of taxes on different consumer's choice: the effect of taxes on consumption by one consumer depends on the effect on other consumers which themselves depend on the effect on other consumers and potentially on the consumer for whom the effect is assessed. Mathematically, this intuition finds its justification from the chain rule for derivatives.

Existence of equilibrium and regularity of economies with externalities is the topic of the following section. The positive aspects of taxes in economies with externalities is then studied. The last section of this paper is devoted to the welfare analysis of economies with externalities and taxes. The main result is the following: there exists an anonymous Pareto improving tax policy. A conclusion precedes appendices on computations and on notations.

2 Economies with externalities

An externality is the effect that one consumer's action has on another consumer's welfare. Externalities mediated through prices are called pecuniary externalities; they are *not*[1] the type of externalities studied here. Shubik (1971) argues that the fundamental difference between pecuniary and physical externalities is that the former can be avoided by forming a coalition while the latter are unavoidable. The focus of this paper is on non-pecuniary externalities.

The setup of an economy with externalities is the same as the one of a classical Walrasian economy *except* for the domain of definition of utility functions. Consumer h's welfare no longer depends solely on consumer h's consumption; it may also depend on other consumer's consumption. Consumers have preferences over $\times_{h \in H} \mathbb{R}^L_{++}$ represented by a utility function $u^h : \mathbb{R}^{HL}_{++} \to \mathbb{R}$.

[1]Geanakoplos and Polemarchakis (1986) show that, due to pecuniary externalities, equilibrium allocations when the asset market is incomplete are constrained Pareto suboptimal.

An economy with externalities $\mathscr{E}$ is defined as a tuple: $\mathscr{E} = \left\langle (u^h, e^h)_{h\in H} \right\rangle$.

2.1 Assumptions

For the sake of unity, all assumptions made at some point in the remaining sections of this paper are gathered in this subsection. Before stating an assumption, we specify for which results the assumption is required.

The following assumption is made throughout the paper.

Assumption 1: Smooth preferences for an economy with externalities.
For every consumer h,

i) u^h is continuous on $\mathbb{R}^{LH}_{++}$ and $\mathcal{C}^2$ on $\mathbb{R}^{LH}_{++}$;

ii) u^h is differentiably strictly monotonically increasing, $D_{x_h}u^h \in \mathbb{R}^L_{++}$;

iii) u^h is differentiably strictly quasiconcave, $D_{x_h}u^h dx = 0 \Rightarrow dx D^2_{x_h} u^h dx < 0$ for every $dx \neq 0$;

iv) for every $x_{-h} \in \mathbb{R}^{L(H-1)}_{+}$ and for every $\bar{u} \in u^h(\mathbb{R}^L_{++}, \mathbb{R}^{L(H-1)}_{+})$, $cl\{x_h \in \mathbb{R}^L_+ : u^h(x_h, x_{-h}) \geq \bar{u}\} \subseteq \mathbb{R}^L_{++}$;

v) $e^h \in \mathbb{R}^L_{++}$.

The following assumption is made for the sections on generic regularity and on the welfare effects of taxes:

Assumption 2: Dominance of own effects.
For every consumer h, for any $x \in \mathbb{R}^{LH}_{++}$ and $v_h \in \mathbb{R}^{LH}$ such that $\sum_h v_h = 0$ and $D_{x_h}u^h v_h = 0$

$$v_h \sum_k D^2_{x_k, x_h} u^h(x_h, x_{-h}) v_k < 0 \text{ whenever } v_h \neq 0.$$

The following two assumptions are made for the section on the welfare effects of taxes:

Assumption 3: Policy: more tools than objectives and existence of trade opportunities.
$L - 1 \geq H$, and $H \geq 2$.

Assumption 4: Diverse marginal external effects.
For every $\alpha \in \mathbb{R}^H$, with $\alpha \neq 0$, for every equilibrium allocation $x \in \mathbb{R}^{HL}_{++}$, the following

matrix EXT has full row rank:

$$EXT(\alpha, x) = \begin{bmatrix} I_{L-1}|0 & \dots & I_{L-1}|0 \\ D_{x_1}u^1(x) & \dots & D_{x_H}u^1(x) \\ \vdots & \ddots & \vdots \\ D_{x_1}u^H(x) & \dots & D_{x_H}u^H(x) \\ \alpha_1 \ D_{x_1}u^1(x) & \dots & \alpha_H \ D_{x_H}u^H(x) \end{bmatrix}.$$

It can be shown that assumption 4 holds for a set of full Lebesgue measure in the space of economies characterized in section 4.2. Intuitively, marginal external effects can be perturbed without affecting equilibrium equations hence ensuring that generically, the matrix $EXT(\alpha, x)$ has full row rank. We could thus dispense with assumption 4. It is left as an assumption because we believe that it sheds light on the mechanism at stake: taxes cause market clearing prices and the equilibrium allocation to adjust; diverse externalities allow for an array of welfare changes rich enough to include a Pareto improvement.

2.1.1 On the assumptions

Assumption 1 is adapted from Debreu (1972) for the differentiable approach to general equilibrium analysis. It guarantees the differentiability of demand functions. Assumption 1 iv) is instrumental in the proof that the natural projection from the equilibrium manifold to the set of endowments is proper.

Assumption 2 is borrowed from Bonnisseau and del Mercato (2008) for the proof of generic regularity. Assumption 2 is also used in the proof of existence of a Pareto improving policy. Its manifestation comes as a reminder that the exercise of evaluating the welfare effect of an intervention in the economy is meaningful only at regular equilibria. Assumption 2 ensures that the external effect on one consumer's marginal utilities is dominated by the effect of her own consumption.

Assumption 3 ensures that there at least as many policy tools as there are policy objectives (Citanna et al (1998) refer to this condition as the Tinbergen principle).

Assumption 4 is required for the proof of generic existence of a Pareto improving policy. The need for assumption 4 arose naturally from the proof of generic existence of a Pareto improving tax policy. It requires that there are diverse external effects. It is comforting that the classical economy without external effects does not satisfy this condition as the First Fundamental Theorem of Welfare Economics assures us of the efficiency of their competitive equilibrium.

Remark: assumption 4 is not vacuously true.
If there are no externalities $D_{x_k}u^h = 0$ for all $k \neq h$, the matrix does not have full row rank for every α. The following non zero vector $\alpha = (0, \dots, 0, -\alpha_1, \dots, -\alpha_H, 1)$ linearly combine the rows to give $0 \in \mathbb{R}^{HL}$.

Remark: results using assumption 4 are not vacuously true.
This example shows that assumption 4 is not impossible to satisfy. Firstly, the matrix of interest is of size $H + L \times HL$; since $H + L < HL$ by assumption 3, it may have full row rank. Secondly, consider an economy with four goods ($L = \{1, 2, 3, 4\}$), two consumers $H = \{1, 2\}$ with respective utility functions u_1 and u_2. For simplicity, assume that the external effects from consumption of commodity 4, the numéraire, are null; that is $D_{x_k^4}u^h = 0$ for every $k \neq h$. After gathering columns with derivative with respect to the numéraire at the end, the matrix[2] $EXT(\alpha, x) =$

$$\begin{bmatrix} I_{L-1} & \dots & I_{L-1} & 0 \\ D_{x_1^{\setminus}} u^1(x) & \dots & D_{x_H^{\setminus}} u^1(x) & D_{x^C} u^H \\ \vdots & \ddots & \vdots & \vdots \\ D_{x_1^{\setminus}} u^H(x) & \dots & D_{x_H^{\setminus}} u^H(x) & D_{x^C} u^H \\ \alpha_1 \ D_{x_1^{\setminus}} u^1(x) & \dots & \alpha_H \ D_{x_H^{\setminus}} u^H(x) & \alpha \otimes D_{x^C} u^H \end{bmatrix}$$

For this example, Assumption 4 requires (computations can be found in the appendix) that the following does *not* hold true:

$$\frac{\frac{\partial u_1}{\partial x_2^2} - \frac{\partial u_1}{\partial x_2^3}}{\frac{\partial u_1}{\partial x_2^2} - \frac{\partial u_1}{\partial x_2^1}} = \frac{\frac{\partial u_2}{\partial x_1^2} - \frac{\partial u_2}{\partial x_1^3}}{\frac{\partial u_2}{\partial x_1^2} - \frac{\partial u_2}{\partial x_1^1}}.$$

This condition involves only external effects and require that they are diverse. The larger is the number of goods relative to the number of types of people, the weaker is the requirement assumption 4 puts on the profile of preferences.

2.2 Equilibrium of an Economy with externalities

Definition: Competitive equilibrium. A competitive equilibrium is a tuple of consumption choices and prices (x, p) such that for every h:

(u^h) $\qquad x_h \in \arg\max_{x \in \mathbb{R}^L_{++}} u^h(x, x_{-h})$ subject to $\qquad p\ (x - e_h) \leq 0$

(M) $\qquad \sum_h x_h = \sum_h e_h$

[2] Where $\alpha \otimes D_{x^C} u^H := (\alpha_1 D_{x_1^C} u^1, \dots, \alpha_H D_{x_H^C} u^H)$.

The First order conditions for the utility maximization problem for consumer h are necessary and sufficient (by assumption 1, smooth preferences) for a maximum; they are:

$$\begin{aligned} D_{x_h} u^h(x_h, x_{-h}) - \lambda_h \; p = 0 \\ p(x_h - e_h) = 0. \end{aligned}$$

For further reference, define Ξ as the set of endogenous variable and let ξ denote an element from it. That is: $((x_h, \lambda_h)_h, p^\backslash) =: \xi \in \Xi := \times_h \big(\mathbb{R}^L_{++} \times \mathbb{R}_{++}\big) \times \mathbb{R}^{L-1}_{++}$.
The assumptions on preferences guarantee that the first order conditions are not only necessary, but also sufficient for a maximum. It is thus immediate that the 0 of the following function constitutes an alternative definition for an equilibrium. $F_e : \Xi \to \times_h \mathbb{R}^{L+1} \times \mathbb{R}^{L-1}$ is defined by:

$$F_e(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\backslash) = \begin{bmatrix} D_{x_1} u^1(x_1, x_{-1}) - \lambda_1 p \\ p(x_1 - e_1) \\ \vdots \\ D_{x_H} u^h(x_H, x_{-H}) - \lambda_H p \\ p(x_H - e_H) \\ \sum_h x_h^\backslash - e_h^\backslash \end{bmatrix}$$

Define $F(\xi, e) := F_e(\xi)$.

We will work with the following equivalent[3] definition:

Definition: Competitive equilibrium. A competitive equilibrium of the economy (e, u) is a tuple of consumption choices, Lagrange multipliers and prices $(x, \lambda, p^\backslash)$ such that: $F_e^C(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\backslash) = 0$.

2.3 Existence of equilibrium of economies with externalities

Results for the existence of equilibria for economies with externalities are well known at least since Laffont and Laroque (1972). The main challenge Laffont and Laroque faced in proving the existence of equilibrium for economies with externalities is the problem of non-convexity production can raise (even if for each level of externality, production sets

[3] It is hoped that the abuse of the definition of equilibrium will not create confusion. The first definition defines an equilibrium (x, p) while the second includes Lagrange multipliers in the definition (x, λ, p). For fixed utility functions in the equivalence class of utility functions representing consumer preferences, Lagrange multipliers are uniquely determined at equilibrium, thus the projection $(x, \lambda, p) \to (x, p)$ gives a bijective relationship between the equilibrium sets corresponding to the respective definitions.

are convex). They overcome this problem with the use of an auxiliary economy whose consumption and production sets are convexified and compactified, the equilibrium of which is proved to also be an equilibrium for the original economy. While Laffont and Laroque (1972) use a classical fixed point argument, recently E. del Mercato (2006) recast the result of Laffont and Laroque (1972) with an argument using homotopies.

2.4 Generic regularity of economies with externalities

Definition: Regular economy. An economy e is regular if it is a regular value of the projection map $\Pi : \ F^{-1}(0) \to \mathbb{R}^{LH}_{++}$ defined by $(x, \lambda, p, e) \mapsto e$.
Let $\mathscr{E}^R \subseteq \mathbb{R}^{LH}_{++}$ denote the set of all regular economies.

As the following example from Bonnisseau and del Mercato (2008) shows, with the standard assumption 1 (smooth preferences) only, it is hopeless to prove generic regularity of economies with externalities in the space of economies parametrized by endowments only.

Example: Siconolfi-Bonnisseau-del Mercato. Let $\epsilon \in \mathbb{R}_{++}$. There are two consumers with utility functions $u_h : \mathbb{R}^2_{++} \times \mathbb{R}^2_{++} \to \mathbb{R}$ defined by:

$$u_1(x_1, x_2) = \ln((1+\epsilon)\ x_1^1 + x_2^1) + x_1^2 + \tfrac{1}{1+\epsilon}\ x_2^2$$

$$u_2(x_2, x_1) = \ln((1+\epsilon)\ x_1^1 + x_2^1) + x_1^2 + \tfrac{1}{1+\epsilon}\ x_2^2$$

and endowments $(e_1, e_2) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_{++}$.
Combining First Order Conditions $D_x u^1(x_1, x_2) = \ \lambda_1 p$ and $D_x u^2(x_2, x_1) = \lambda_2 p$, at an equilibrium:

$$MRS_1 := \frac{\frac{\partial u^1(x_1,x_2)}{\partial x_1^1}}{\frac{\partial u^1(x_1,x_2)}{\partial x_1^2}} = \frac{1+\epsilon}{(1+\epsilon)x_1^1 + x_2^1} = \frac{\frac{\partial u^2(x_2,x_1)}{\partial x_2^1}}{\frac{\partial u^2(x_2,x_1)}{\partial x_2^2}} =: MRS_2$$

Note that this equation is well defined since denominators are strictly positive. MRS_h denotes the marginal rate of substitution of individual h. The marginal rates of substitution are equalized at any allocation (x_1, x_2). ϵ introduces an asymmetry between the two consumers. Using market clearing $x_1 + x_2 = e_1 + e_2$, the marginal rate of substitutions become:

$$MRS_1 = MRS_2 = \frac{1+\epsilon}{e1_1 + e1_2 + \epsilon(z_1^1 + e_1^1)}$$

Since $(e_1, e_2) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_{++}$, there exists an open ball $U_1 \in \mathbb{R}$ around zero such that for every $z^1_1 \in U_1$, there is an equilibrium $\big(p, (e^1_1 + z^1_1, e^2_1 - p_1 z^1_1), (e^1_2 - z^1_1, e^2_2 + p_1 z^1_1)\big)$ with prices $p_2 = 1$ and $p_1 = MRS_h(z^1_1)$. That is, for every endowment, there is a continuum of equilibria. Hence, every economy is *non* regular in this example.

This example epitomizes the new challenges raised by externalities for the issue of determinacy. The external effect on consumer 2's marginal utilities dominates the effect of her own consumption. As it will emerge from the proof of generic regularity, an additional condition on utility functions is needed; this is assumption 2, dominance of the effect of own consumption. The utility function of consumer 2, in this example, does not satisfy assumption 2, as the following matrices make clear:

$$D_{x_1} u^1 = \begin{bmatrix} -\frac{1+\epsilon}{(1+\epsilon)x^1_1 + x^1_2} \\ 1 \end{bmatrix} \qquad D_{x_2} u^2 = \begin{bmatrix} -\frac{1}{(1+\epsilon)x^1_1 + x^1_2} \\ \frac{1}{1+\epsilon} \end{bmatrix}$$

$$D^2_{x_1,x_2} u^1 = \begin{bmatrix} -\frac{1}{((1+\epsilon)x^1_1 + x^1_2)^2} & 0 \\ 0 & 0 \end{bmatrix} \qquad D^2_{x_1,x_2} u^2 = \begin{bmatrix} -\frac{1+\epsilon}{((1+\epsilon)x^1_1 + x^1_2)^2} & 0 \\ 0 & 0 \end{bmatrix}$$

The following result was recently proved by Bonnisseau and del Mercato (2008). The only minor refinement the proof below offers is a more parsimonious use of perturbations of endowment. This is in anticipation of the need for sources of perturbation independent of the ones used for regularity in order to prove further results (e.g. full trade at equilibrium). The endowment of one individual is perturbed in all commodities, while for all other consumers, only the numéraire commodity is perturbed. Bonnisseau and del Mercato (2008) did not need to perturb the numéraire *only* for all but one consumer; indeed regularity was the central argument of their contribution. This paper uses their result as a stepping stone to further results on the analysis of welfare.

Assumptions 1) smooth preferences and 2) dominance of own effects are made for all the results to come.

Lemma. 0 *is a regular value of* F.

Proof. The proof strategy is to show that for each $\xi \in F^{-1}(0)$, $D_\xi F$ has full rank, it is shown that for any $\Delta = (\delta_x, \delta_\lambda, \delta_{p^\backslash}) \in \mathbb{R}^{HL+H+L-1}$, $\qquad \Delta D_{\xi,e}F = 0 \Longrightarrow \Delta = 0$.

$$D_{\xi,e}F(\xi,e) = \begin{bmatrix} D^2_{x_1}u^1 & p & \dots & D^2_{x_1,x_H}u^1 & 0 & 0 & 0 & \dots & 0 & 0 & -\lambda_1 \frac{I_{L-1}}{0} \\ p^T & 0 & \dots & 0 & 0 & -p^\backslash & 1 & \dots & 0 & 0 & (x_1^\backslash - e_1^\backslash)^T \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & 0 & \dots & -1 & 0 & \vdots \\ D^2_{x_H,x_1}u^H & 0 & \dots & D^2_{x_H}u^H & p & 0 & 0 & \dots & 0 & 0 & -\lambda_2 \frac{I_{L-1}}{0} \\ 0 & 0 & \dots & p^T & 0 & 0 & 0 & \dots & 0 & -1 & (x_2^\backslash - e_2^\backslash)^T \\ I_{L-1}|0 & 0 & \dots & I_{L-1}|0 & 0 & I_{L-1} & 0 & \dots & 0 & 0 & 0 \end{bmatrix}$$

Variables with respect to which derivatives are taken for the corresponding column:

$x_1 \quad \lambda_1 \quad x_H \quad \lambda_H \quad e_1^\backslash \quad e_1^C \quad e_h^C \quad e_H^C \quad p^\backslash$

The system of equations resulting from $\Delta\ D_{\xi,e}F = 0$ is:

$(1) \forall k \in H, \qquad \sum_h \delta_{x_h} D^2_{x_k,x_h}u^h + \delta_{\lambda_h} p + \delta_{p^\backslash} I_{L-1}|0 = 0$

$(2) \forall k \in H, \qquad \delta_{x_k} p = 0$

$(3a)$[4] $\qquad -\delta_{\lambda_1} p^\backslash - \delta_{p^\backslash} I_{L-1} = 0$

$(3b) \forall k, \quad -\delta_{\lambda_k} = 0$

$(4) \qquad \sum_h \delta_{x_h^\backslash} \lambda_h + \sum_h \delta_{\lambda_h} (x_h^\backslash - e_h^\backslash) = 0$

From $(3b)$, we get that, for all $h \in H$, $\delta_{\lambda_h} = 0$. In turn, with $\delta_{\lambda_1} = 0$, $(3a)$ implies that $\delta_{p^\backslash} = 0$.

$(1.1) \forall k \in H, \qquad \sum_{h \in H} \delta_{x_h}\ D^2_{x_k,x_h}u^h = 0$

$(2.1) \forall k, \qquad \delta_{x_k}\ p = 0$

$(4.1) \quad \sum_h \delta_{x_h^\backslash}\ \lambda_h = 0$

For all h, at an equilibrium $D_{x_h}u^h - \lambda_h\ p = 0$. From (2), $D_{x_h}u^h\ \delta_{x_h} - \lambda_h\ p\ \delta_{x_h} = D_{x_h}u^h\ \delta_{x_h} = 0$.

Define $v_h = \delta_{x_h}\ \lambda_h$.

From (2.1), $\delta_{x_h^C} = -\delta_{x_h^\backslash}\ p^\backslash$ therefore $\sum_h \lambda_h \delta_{x_h^C} = -p^\backslash \sum_h \lambda_h \delta_{x_h^\backslash} = 0$.

Therefore, from (4.1),and the line above $\sum_h v^h = 0$.

After post-multiplication by v_k and summing over k, (1.1) becomes:

[4] This additional equation involving only δ_{λ_1} and not δ_{λ_h} reminds us that all commodities with which consumer 1 is endowed are perturbed whereas commodity C is perturbed for all consumers. As in Bonnisseau del Mercato, the proof goes through if all endowments of all consumers are perturbed. This provident choice will prove useful when proving that generically, every trader trades every commodity.

$\sum_k \sum_h \delta_{x_h} \; D^2_{x_k,x_h} u^h v_k = 0$;
after interchanging the order of (finite) summations, it reads as follows:
$\sum_h \frac{1}{\lambda_h} \; v_h \sum_k \; D^2_{x_k,x_h} u^h v_k = 0$.
Since $\sum_h v^h = 0$ and $0 = D_{x_h} u^h \; \delta_{x_h} = D_{x_h} u^h \; v_h$, assumption 3 (dominance of own effects) guarantees that $v_h = 0$ for every h; furthermore, since $\lambda_h > 0, \quad \delta_{x_h} = 0$.
$\Delta = (\delta_{x_h}, \delta_{\lambda_h}, \delta_{p\backslash}) = 0$. □

Theorem 1. *The set of regular classical economies with externalities $\mathscr{E}^R$ is open and has full Lebesgue measure.*

Proof. From the lemma just proved, 0 is a regular value of $F : \Xi \times \mathbb{R}^{LH}_{++} \to \times_h \mathbb{R}^{L+1} \times \mathbb{R}^{L-1}$. By the transversality theorem, there exists a subset $\mathscr{E}^R$ of $\mathbb{R}^L H_{++}$ that has full Lebesgue measure $\lambda_L H$ such that:

$$\text{for every } e \in \mathscr{E}^R, \qquad 0 \text{ is a regular value of } F_e : \Xi \to \times_h \mathbb{R}^{L+1} \times \mathbb{R}^{L-1}$$

In other words, the set of regular economies has full Lebesgue measure.
To show that $\mathscr{E}^R$ is open, the argument made for the case without externalities applies. It is repeated here. The set of regular values is the complement to the set of critical values. A value is critical for a map if there is at least one point in the preimage of the value at which the (pushforward of the map is not surjective. In other words; $e \in \mathbb{R}^{LH}_{++}$ is a critical value of π if there exists $(p, e) \in \pi^{-1}(e)$ such that the Jacobian of π evaluated at (p, e) does not have full row rank. A sufficient condition is that any square matrix of size 'the number of rows' extracted from the Jacobian has determinant 0. There are finitely many such matrices; since the determinant is continuous and $\{0\}$ is closed, the set of critical points is the finite union of closed sets. □

Proposition. *The restricted projection map* Π : *is proper.*

Proof. The proof is identical to the one for an economy without externalities (cf. Balasko (2009)). □

The set of equilibria is thus finite.

F satisfies the conditions for the Implicit Function theorem to apply. Hence endogenous variables depend smoothly on endowment at equilibria ξ of regular economies.

3 Economies with externalities and taxes

The policy tools are restricted to be anonymous. Such a restriction stems from the informational asymmetry between the policy maker and consumers. Even if a policy that makes everybody better off exists, it may not be in the interest of consumers to truthfully reveal their preferences to the policy authority. Indeed, there may exists more than one Pareto improving policy and these policy may favour different consumers to varying extent. Therefore, even in the presence of Pareto improving policies, the asymmetry of information may constrain the government in the implementation of a policy.

The policy tools are commodity taxes $t \in \mathbb{R}^{L-1}$ and a lump sum transfer $\tau \in \mathbb{R}$ of equal amount to consumers. Both t and τ are anonymous. A commodity l is taxed when $t_l > 0$ and subsidized when $t_l < 0$. The reason for taxing only $L-1$ commodities is motivated by the following argument: assume there are only two goods; if the numéraire is taxed at rate t_C and the other good is taxed at rate t_l, the relative after-tax prices faced by a consumer who buys (sells) the numéraire and sells (buys) commodity l are $\frac{p_l+t_l}{1}$ and $\frac{p_l}{1+t_C}$ respectively. If $(1+t_C)(p_l+t_l) = p_l$, the after-tax relative price of commodity l equals the pre-tax relative price.

A government policy $(t,\tau) \in \mathbb{R}^{L-1} \times \mathbb{R}$ is subject to a budget balance constraint Γ.

$$\Gamma : \Xi \times \mathbb{R}^{L-1} \times \mathbb{R}$$

$$\Gamma(\xi,t,\tau) = \sum_{l=1}^{L-1} \sum_{h \in H} t_l \; z^h_{l,+} - H\tau = 0$$

where the following notation is used: $z^h_{l,+} := \max\{0, z^h_l\}$. For future reference, $z^h_{l,-} := -\min\{0, z^h_l\}$.

Definition: Competitive equilibrium with taxes. $(z,p,t,\tau) \in \times_{h \in H}\mathbb{R}^L \times \mathbb{R}^L_{++} \times \mathbb{R}^L \times \mathbb{R}$ is a competitive equilibrium with taxes if:

1) Consumers maximize utility:

$$z^h \in \arg\max_z u^h\big((e_h + z), (e_{-h} + z_{-h})\big) \quad \text{subject to:} \quad (p+t)\; z_+ - p\; z_- \leq \tau;$$

2) Markets clear:

$$\sum_{h \in H} z^h = 0;$$

3) Government budget balances:

$$\sum_{l=1}^{L} \sum_{h \in H} t_l \; z^h_{l,+} = H\tau.$$

3.1 Existence of equilibrium with taxes and full trade

Taxes and subsidies complicate the differentiable approach to equilibrium analysis: the budget set becomes kinked and in the presence of subsidies, it may not even be convex. The kink poses problem for the differentiability of demand. Non-convexities challenge the use of first and second order conditions for maximization. Furthermore, demand may not be single valued. This problem prevents us from doing the analysis directly from an equilibrium with taxes and transfers. That is, considering an equilibrium of an economy with externalities but no taxes, the aim is to show that there exists an equilibrium with taxes that Pareto dominates the equilibrium without taxes.

3.1.1 Generic full trade at equilibrium

Generically, consumers do not consume their endowment at equilibrium and therefore, a small enough tax will not pose problem for the differentiable approach to equilibrium analysis.

Proposition[5]**.** *The set of regular classical economies with externalities and full trade* $\mathscr{E}^{FR}$ *is open and has full Lebesgue measure.*

Proof. Let $\hat{F}_{h,l,e} : \Xi \to \times_h \mathbb{R}^{L+1} \times \mathbb{R}^{L-1} \times \mathbb{R}$ be defined by:

$$\hat{F}_{h,l;e}(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\setminus) = \begin{bmatrix} F_e(\xi) \\ x^l_h - e^l_h \end{bmatrix}$$

The map $\hat{F}_{h,l;e}$ has the same domain as the map F_e defined above. It differs from F_e only in that it maps to a space with 1 more dimension as it is augmented by $x^l_h - e^l_h \in \mathbb{R}$. As usual, define, the map $\hat{F}_{h,l}(\xi, e) := \hat{F}_{h,l;e}(\xi)$.

$$D_{\xi,e}\hat{F}_{h,l}(\xi, p^\setminus, e) = \begin{bmatrix} D_{\xi,e}\hat{F} \\ D_{\xi,e}(x^l_h - e^h_l) \end{bmatrix}$$

[5]The proof is an adaptation to the approach using First Order Conditions of a proof in Geanakoplos and Polemarchakis (2008).

Claim: 0 is a regular value of $\hat{F_{hl}}$.

$$D_{\xi,e}\hat{F}_{h,l}(\xi) = \begin{bmatrix} D_{\xi,e}\hat{F}(\xi) \\ D_{\xi,e}(x_h^l - e_h^l) \end{bmatrix}$$

Let $h \neq 1$ and $l \neq C$ We know from the proof that 0 is a regular value of F that the matrix $D_{\xi,e}\hat{F}(\xi)$ has full row rank. It thus suffices to show that we can perturb the last row without perturbing $D_{\xi,e}\hat{F}(\xi)$. Thanks to the parsimonious use of perturbation parameters in the proof of generic regularity, e_h^l can be used to perturbed the last row independently of $D_{\xi,e}\hat{F}(\xi)$. Since e_l^h was not used in the proof of generic regularity, the effect the perturbation of e_h^l has on $D_{\xi,e}\hat{F}(\xi)$ can be undone by perturbing the variables perturbed in the proof of regularity. The claim is thus true for $h \neq 1$ and $l \neq C$: 0 is a regular value of $\hat{F_{hl}}$. By the transversality theorem, there exists a subset $\mathscr{E}_{lh}^R$ of $\mathbb{R}^L H_{++}$ that has full Lebesgue measure such that: for every $e \in \mathscr{E}_{lh}^R$, 0 is a regular value of $\hat{F}_{h,l,e}$. The set of regular economies for which individual h trades commodity l has full Lebesgue measure.

The choice of consumer 1 and good C in the proof of generic regularity was arbitrary. The proof can be done with any other arbitrary choice of consumer and good. Hence, this procedure can be repeated for every h, and every l. There are finitely many consumers and finitely many goods The intersection of finitely many sets of full measure is a set of full measure. Define $\mathcal{E}^{RF} = \bigcap_{\substack{h\in H \\ l\in L}} \mathscr{E}_{lh}^R$. The intersection of finitely many sets of full Lebesgue measure is itself a set of full Lebesgue measure; $\mathcal{E}^{RF}$ has full Lebesgue measure. □

3.1.2 Existence of equilibrium with -sufficiently small- taxes

For[6] economies belonging to the generic set of regular economies with full trade $\mathcal{E}^{RF}$, the kink and potential non-convexities due to taxes and subsidies are not problematic for small enough taxes.

As previously mentioned, to accommodate the challenges posed by taxes to the differentiable approach to the study of competitive markets, the analysis is conducted locally at a regular equilibrium with full trade. In an economy with consumers with strictly quasi-concave utility functions in which *every* individual trades *every* commodities at equilibrium, infinitesimal taxes do not affect the differentiability of demand.

Define personalized taxes for every h, l as follows: $t_h^l : \cup_{(e,u)\in\mathcal{E}^{RF}} F_{e,u}^{-1}(0) \to \mathbb{R}$.

[6]This subsection is borrowed from Geanakoplos and Polemarchakis (2008).

At an equilibrium with full trade $\xi = ((x_h, \lambda_h)_h, p^\setminus) \in F_{e,u}^{-1}(0)$,

$$t_h^l(\xi) = \begin{cases} t_h^l & \text{if } x_h^l - e_h^l > 0 \\ 0 & \text{if } x_h^l - e_h^l < 0. \end{cases}$$

The equilibria of an economy with taxes are elements of the preimage of 0 from the following function:

$$F_e^{tax} : \Xi \times \mathbb{R}^{L-1} \times \mathbb{R} \to \mathbb{R}^{\dim \Xi} \times \mathbb{R}$$

$$F_e^{tax}(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\setminus, t, \tau) = \begin{bmatrix} D_{x_1} u^1(x_1, x_{-1}) - \lambda_1(p + t_1) \\ (p + t_1)(x_1 - e_1) - \tau \\ \vdots \\ D_{x_H} u^h(x_H, x_{-H}) - \lambda_H(p + t_H) \\ (p + t_H)(x_H - e_H) - \tau \\ \sum_h x_h^\setminus - e_h^\setminus \\ \sum_{l=1}^{L-1} \sum_{h \in H} t_h^l \; z_{l,+}^h - H\tau \end{bmatrix}$$

$$D_{\xi,e} F_e^{tax} = \begin{bmatrix} D_\xi F & D_t F & D_\tau F \\ D_\xi \Gamma & D_t \Gamma & D_\tau \Gamma \end{bmatrix}$$

Proposition. *For every regular economy with full trade $e \in \mathcal{E}^{FR}$, there exists a open subsets $\mathcal{U}_t \times \mathcal{U}_\tau \subset \mathbb{R}^{L-1} \times \mathbb{R}$, $\mathcal{U}_\Xi \subset \Xi$ and a differentiable function $\xi : \mathcal{U}_t \times \mathcal{U}_\tau \to \mathcal{U}_\Xi$ such that:*

$$F_{e,u}^{tax}(\xi, t, \tau) = 0 \iff \xi = \xi(t, \tau)$$

Proof. cf. Geanakoplos and Polemarchakis (2008). The proof consists in an application of the Implicit Function Theorem. □

4 The welfare effect of taxes

4.1 Welfare criteria

Definition: Pareto optimality. An allocation $x \in \mathbb{R}_{++}^{LH}$ is Pareto optimal if there does not exist another feasible allocation $\bar{x} \in \mathbb{R}_{++}^{LH}$ such that:

- for every $h \in H$, $\quad u^h(\bar{x}) \geq u^h(x)$

- for at least one $k \in H$, $\quad u^k(\bar{x}) > u^k(x)$.

Definition: Constrained Pareto optimality. An allocation $x \in \mathbb{R}^{LH}_{++}$ is constrained Pareto optimal if there does not exist a tax policy $(t, \tau) \in \mathbb{R}^L \times \mathbb{R}$ and an equilibrium with taxes $(\bar{x}, \lambda, p, t, \tau)$ such that $\bar{x}$ Pareto dominates x.

The concept of constrained Pareto optimality differs from Pareto optimality in that the policy maker is constrained to allocate resources through markets. The requirement for the policy to be anonymous recognizes the informational constraint the policy authority faces for the implementation of the policy.

The main result of this paper is that, generically, equilibrium of economies with externalities are *constrained* Pareto suboptimal. Externalities that are not additively separable raise the challenge that the effects of taxes on one consumer's demand may depend on the effects of taxes on another consumer which in turn may depend on the effect assessed. Geanakoplos and Polemarchakis (2008) based their analysis on excess demands; while their approach lays bare the role of demand theory and hence convey intuition, these potential feedbacks of externalities pose problem for the extension of their analysis to non additively separable externalities. This challenge motivates a different approach using First Order Conditions. The analysis of First Order Conditions that characterize equilibria is conducted at a more primitive level and thus eschews these potentially cyclical feedback effects.

4.2 Space of economies

This section introduces utility perturbations as used by Citanna et al (1998) and Villanacci et al (2002) and expands perturbations to external effects.
The set of equilibria for an economy $e \in \mathcal{E}^{RF}$ is finite. Index the equilibria by $m = 1, \ldots, M$; i.e. $F_e^{-1}(0) = \{\xi^1, \ldots, \xi^M\}$ As the example in section 6) of Geanakoplos and Polemarchakis (2008) prove, the result of *constrained* Pareto optimality of economies with external effects cannot hold for any specification of external effects; it may well hold, however, for a generic set of external effects and endowments.
$F^{-1}(0)$ is a smooth manifold and there exists disjoints open sets $U_m \in F^{-1}(0)$ such that $\xi^m \in U_m$ for every $m = 1, \ldots, M$. Projections are open maps, therefore, the projection $\xi^m \to x^m$ of U_M is an open set; denote it U_m. Lee (2003) Proposition 2.26 guarantees the existence of bump functions b^m with support U_m for the closed set $\{x^m\}$, so $b_m(x^m) = 1$.

For this section, the utility functions are perturbed by symmetric matrices denoted A_h for own effects and A_i for external effects; $\tilde{u}_h(x_h, x_{-h}, A_h) =$

$$u_h(x_h, x_{-h}) + \frac{1}{2}\sum_m b_m(x^m)[(x_h - x_h^m)^T \, A_h^h \, (x_h - x_h^m)] + \sum_m b_m(x^m)[(x_h - x_h^m) \, A_i^h \, (x_i - x_i^m)]$$

For A_{-h} and A_h sufficiently small, $\tilde{u}_h$ satisfies the assumption 1 if u_h does. Define $\mathcal{A}$ the set of profiles of $L \times L$ symmetric matrices. such that $\tilde{u}_h$ satisfies assumption 1 (smooth preferences) for every h; hence $0 \in \mathcal{A}$. $\mathcal{A}$ can be seen as a finite dimensional submanifold of the space of utility functions satisfying assumption 1.

The design of the utility perturbations is such that the utility perturbations does not affect equilibria. Indeed, the slope of indifference curves, which is of primordial importance for the specification of an equilibrium is not affected by the utility perturbations; at equilibrium, only the curvature of the indifference curve is perturbed.

Notation:

From now on, utilities are always understood to be perturbed. To keep the notation simple, the following abuse of notation is made: u_h refers to $\tilde{u}_h$. Note that $0 \in \mathcal{A}$ so the economies with fixed u considered so far are a special case of the ones we consider now.

Economies are now parametrized by endowments and utility functions; $(e, A) \in \mathbb{R}^{HL}_{++} \times \mathcal{A}$.

Remark: My attempt at proving generic constrained Pareto suboptimality of equilibrium allocation of economies with externalities for a generic set in the space of external effects and endowments led to the weaker result of genericity in the space of external effects, endowments *and* own effects. The equations obtained hinted at the numéraire commodity, which is not taxed, as the source of inconclusiveness. The motivation to perturb own effects comes from the following conjecture: assumption 4 does not require every consumers' every actions to have an effect on all other consumers; some economies with as few as two consumers whose actions have an effect on all consumers may be constrained Pareto suboptimal. However, no natural condition on the external effect arose from these proofs, except assumption 4. Suppose only two consumers actions have external effects on every other consumers. Even though every individual trades every commodity, the two consumers with external effects may be selling all commodities but the numéraire. If in addition, the effect of taxes on other consumers whose demand has no external effects compensate one another in the aggregate, taxes would not effect any price[7] change. The two consumers whose demand has externalities would thus face the same price after tax – as they *sell* the commodities taxed. Taxes would thus not induce any change in the external effects. The intuition behind perturbing own effects rests on the fact that the possibility that the effect of taxes on individual demands compensate one another in the aggregate requires a peculiar specification of utility functions that are of measure 0 in the space of own effects.

[7]Tax excluded.

4.3 Existence of a Pareto improving tax policy

The methodology developed in Citanna-Kajii-Villanacci (1998) is adopted. Before proceeding, the functions used in the proof are recalled. These functions are not the same as the one used so far. Perturbed utilities are used. For example, the first line of the function defined below should read: $D_{x_1}\tilde{u}^1(x_1, x_{-1}) - \lambda_1 p$. Recall that A is always understood to be an element of $\mathcal{A}$ and that $0 \in \mathcal{A}$. When $A = 0$ the economy coincides with the original economy.

The function characterizing equilibrium without taxes is:
$F_{e,A} : \Xi \to \times_h \mathbb{R}^{L+1} \times \mathbb{R}^{L-1}$ is defined by:

$$F_{e,A}(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\setminus) = \begin{bmatrix} D_{x_1}u^1(x_1, x_{-1}) - \lambda_1 p \\ p(x_1 - e_1) \\ \vdots \\ D_{x_H}u^h(x_H, x_{-H}) - \lambda_H p \\ p(x_H - e_H) \\ \sum_h x_h^\setminus - e_h^\setminus \end{bmatrix}$$

Define $F(\xi, e, A) := F_{e,A}(\xi)$, with as usual, the domain of F being the domain of $F_{e,A}$ expanded accordingly; the domain of F has become $\Xi \times \mathbb{R}^{LH}_{++} \times \mathcal{A}$.

Function characterizing equilibrium with taxes:
$F^{tax}_{e,A} : \Xi \times \mathbb{R}^{L-1} \times \mathbb{R} \to \mathbb{R}^{\dim \Xi} \times \mathbb{R}$ is defined by:

$$F^{tax}_{e,A}(x_1, \lambda_1, \ldots, x_H, \lambda_H, p^\setminus, t, \tau) = \begin{bmatrix} D_{x_1}u^1(x_1, x_{-1}) - \lambda_1(p + t_1) \\ (p + t_1)(x_1 - e_1) - \tau \\ \vdots \\ D_{x_H}u^h(x_H, x_{-H}) - \lambda_H(p + t_H) \\ (p + t_H)(x_H - e_H) - \tau \\ \sum_h x_h^\setminus - e_h^\setminus \\ \sum_{l=1}^{L-1} \sum_{h \in H} t_h^l \; z_{l,+}^h - H\tau \end{bmatrix}$$

Define $F^{tax}(\xi, e, A) := F^{tax}_{e,A}(\xi)$.

Function assessing locally the welfare effect at outcome [8] ξ of a policy intervention subject to the constraint that the planner is facing:

[8] The matrices are evaluated at ξ.

$\Phi_{e,A} : \Xi \times \mathcal{S}^{H+\dim\Xi} \to \mathbb{R}^H \times \mathbb{R}^{\dim\Xi} \times \mathbb{R}$

$$\Phi_{e,A}(\xi,\Delta)|_{(\xi,0,0)} = \Delta \begin{bmatrix} D_\xi U & D_t U \\ D_\xi F^{tax} & D_t F^{tax} \end{bmatrix} = \Delta \begin{bmatrix} D_\xi U & 0 \\ D_\xi F^{tax} & D_t F^{tax} \end{bmatrix}$$

where

$$\begin{bmatrix} D_{\xi,t} U \\ D_{\xi,t} F^{tax} \end{bmatrix} = \begin{bmatrix}
D_{x_1} u^{1^T} & 0 & \dots & D_{x_H} u^{1^T} & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots \\
D_{x_1} u^{H^T} & 0 & \dots & D_{x_H} u^{H^T} & 0 & 0 & 0 & 0 \\
 & & & & & & & \\
D^2_{x_1} u^1 & p & \dots & D^2_{x_H,x_1} u^1 & 0 & -\lambda_1 \frac{I_{L-1}}{0} & *_{\lambda_1} & 0 \\
p^T & 0 & \dots & 0 & 0 & (x_1^\backslash - e_1^\backslash)^T & *_{(x_1^\backslash - e_1^\backslash)} & -1 \\
\vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots \\
D^2_{x_1,x_H} u^H & 0 & \dots & D^2_{x_H} u^H & p & -\lambda_H \frac{I_{L-1}}{0} & *_{\lambda_H} & 0 \\
0 & 0 & \dots & p^T & 0 & (x_H^\backslash - e_H^\backslash)^T & *_{(x_H^\backslash - e_H^\backslash)} & -1 \\
I_{L-1}|0 & 0 & \dots & I_{L-1}|0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & *_{z^\backslash} & -H
\end{bmatrix}$$

Variables with respect to which derivatives are taken for the corresponding column:

$x_1 \quad \lambda_1 \quad \dots \quad x_H \quad \lambda_H \quad p^\backslash \quad t \quad \tau$

Denote $\Delta = (\delta_{u^1}, \dots, \delta_{u^H}, \delta_{x_h}, \delta_{\lambda_h}, \delta_{p^\backslash}, \delta_\tau) \in \mathbb{R}^{H+\dim\Xi+1}$ an element of the circle[9] $\mathcal{S}^{H+\dim\Xi}$. The index of the elements of Δ are only a reminder of the dimension of the element indexed. Therefore:

$$\Phi_{e,u}(\xi,\Delta)|_{(\xi,0,0)} = \begin{bmatrix}
\vdots \\
\sum_h \delta_{u^h} D_{x_k} u^h + \sum_h \delta_{x_h} D^2_{x_h,x_k} u^h + \delta_{\lambda_k}\ p + \delta_{p^\backslash} I_{L-1}|0 \\
\vdots \\
\delta_{x_k}\ p \\
\vdots \\
-\sum_h \lambda_h I_{L-1}|0\ \delta_{x_h} + \sum_h \delta_{\lambda_h}(x_h^\backslash - e_h^\backslash) \\
\sum_h \delta_{x_h} *_{\lambda_h} + \sum_h \delta_{\lambda_h} *_{(x_h^\backslash - e_h^\backslash)} + \delta_\tau *_{z^\backslash} \\
-\sum_h \delta_{\lambda_h} - \delta_\tau H
\end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Define $\Phi(\xi, t, \tau, e, A) := \Phi_{e,A}(\xi, t, \tau, \Delta)$.

[9] A circle in $\mathbb{R}^p$ is a manifold, which has dimension $p-1$.

After grouping lines and columns, $[D_\Delta \Phi \;\; D_A \Phi]|_{(\xi,0,0)} =$

$$\begin{bmatrix}
D_{x_1}u^1 & \dots & D_{x_1}u^H & D^2_{x_1}u^1 & \dots & D^2_{x_1,x_H}u^H & p & \dots & 0 & \frac{I_{L-1}}{0} & 0 & \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & N(\delta_x) \\
D_{x_H}u^1 & \dots & D_{x_H}u^H & D^2_{x_H,x_1}u^1 & \dots & D^2_{x_H}u^H & 0 & \dots & p & \frac{I_{L-1}}{0} & 0 & \\
 & & & & & & & & & & & \\
0 & \dots & 0 & p^T & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\
0 & \dots & 0 & 0 & \dots & p^T & 0 & \dots & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & \\
0 & \dots & 0 & -\lambda_1 I_{L-1}|0 & \dots & -\lambda_H I_{L-1}|0 & (x_1^{\backslash} - e_1^{\backslash}) & \dots & (x_H^{\backslash} - e_H^{\backslash}) & 0 & 0 & 0 \\
0 & \dots & 0 & *_{\lambda_1} & \dots & *_{\lambda_H} & *_{(x_1^{\backslash}-e_1^{\backslash})} & \dots & *_{(x_H^{\backslash}-e_H^{\backslash})} & 0 & *_{z^{\backslash}} & 0 \\
0 & \dots & 0 & 0 & \dots & 0 & -1 & \dots & -1 & 0 & -H & 0
\end{bmatrix}$$

where
(define, $\phi^k(A) := \sum_h \delta_{u^h} D_{x_k} u^h \; + \sum_h \delta_{x_h}[D^2_{x_h,x_k} u_h + A^h_k] \; + \delta_{\lambda_k}(p + t_k) \; + \delta_{p^{\backslash}} I_{L-1}|0)$

$$N(\delta_x) = \begin{bmatrix}
D_{A_1^1}\phi^1 & D_{A_1^2}\phi^1 & D_{A_1^3}\phi^1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots & D_{A_2^1}\phi^2 & D_{A_2^2}\phi^2 & D_{A_2^3}\phi^2 & \dots & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & D_{A_3^1}\phi^3 & D_{A_3^2}\phi^3 & D_{A_3^3}\phi^3 & \dots \\
\vdots & & & & & & & & & & &
\end{bmatrix}$$

$$= \begin{bmatrix}
[\delta_{x_1}] & [\delta_{x_2}] & [\delta_{x_3}] & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots & [\delta_{x_1}] & [\delta_{x_2}] & [\delta_{x_3}] & \dots & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & [\delta_{x_1}] & [\delta_{x_2}] & [\delta_{x_3}] & \dots \\
\vdots & & & & & & & & & & &
\end{bmatrix}$$

where

$$[\delta_{x_h}] = \begin{bmatrix}
\delta_{x_h^1} & \delta_{x_h^2} & \delta_{x_h^3} & \delta_{x_h^4} & \dots \\
0 & \delta_{x_h^1} & 0 & \delta_{x_h^2} & \dots \\
0 & 0 & \delta_{x_1^h} & 0 & \dots \\
\vdots & & & &
\end{bmatrix}$$

$$*^T_{\lambda_H} = D_t[D_{x_h} u^h(x_h, x_{-h}) - \lambda_h(p + t_h)] = -D_t \; [\lambda_h \; \frac{t_h}{0}]$$

$$*^T_{(x_h^{\backslash} - e_h^{\backslash})} = D_t[(p + t_h)(x_h - e_h) - \tau] = D_t \; [t_h(x_h^{\backslash} - e_h^{\backslash})] = z^{\backslash}_{h+}$$

$$*^T_{z\backslash} = D_t[\sum_{l=1}^{L-1}\sum_{h\in H} t_h^l\ z_{l,+}^h - H\tau] = D_t\ [\sum_{l=1}^{L-1}\sum_{h\in H} t_h^l\ z_{l,+}^h] = \sum_h\ z_{h+}^{\backslash}$$

Note that if $\delta_{x_h} \neq 0$ for at least two $h \in H$, the matrix $N(\delta_x)$ has full row rank HL.

Function assessing locally the welfare effect *at equilibrium* of a policy intervention subject to the constraint that the planner is facing:
$G_{e,A} : \Xi \times \mathcal{S}^{H+\dim\Xi} \to \mathbb{R}^{\dim\Xi} \times \mathbb{R}^H \times \mathbb{R}^{\dim\Xi} \times \mathbb{R}$

$$G_{e,A}(\xi,\Delta) = \begin{bmatrix} F_{e,A}(\xi) \\ \Phi_{e,A}(\xi,\Delta) \end{bmatrix}$$

Define $G(\xi,\Delta,e,A) := G_{e,A}(\xi,\Delta)$.

If there exists (ξ,Δ) such that $G_{e,A}(\xi,\Delta) = 0$, then ξ is an equilibrium ($F_{e,A}(\xi) = 0$) that is constrained Pareto optimal ($\Phi_{e,A}(\xi,\Delta) = 0$). At least two interpretations can be given to $\Phi_{e,A}(\xi,\Delta) = 0$. Kajii (1992)'s optimization approach obtains analogous conditions from the maximization of a vector valued function of utility functions subject to the constraint that the policy maker is facing, in our case, F^{tax}. This approach gives the elements of Δ the interpretation of the Lagrange multipliers of the constrained optimization. Citanna et al (1998) offer another interpretation based on linear algebra. If the matrix $[D_{\xi,t}U^T \quad D_{\xi,t}F^{tax^T}]^T$ has full row rank, there exists a $\Delta = (\delta_u, \delta_\xi)$ such that everybody is better off $\delta_u D_{\xi,t}U = 1$ at the allocation of an equilibrium with taxes $\delta_\xi\ D_{\xi,t}F^{tax} = 0$.
To prove that $G_{e,A}^{-1}(0) = \emptyset$, it suffices to prove that 0 is a regular value of $G_{e,A}$. Indeed, the Regular Value theorem then guarantees that $G^{-1}(0)$ is a regular submanifold of dimension 'dimension of domain - dimension of range' $= (\dim\,\Xi\ +\ H + \dim\,\Xi) - (\dim\,\Xi + H + \dim\,\Xi + 1) < 0$.

When it is not possible to prove a result for all economies (e,A), the Transversality theorem may allow for a proof that the result holds for almost all economies: if 0 is a regular value of G, then for almost all $(e,A) \in \mathbb{R}^{HL}_{++} \times \mathcal{A}$, 0 is a regular value of $G_{(e,A)}$.

For the following result, economies considered satisfy assumptions 1 to 4.

Theorem. *The set of regular economies with full trade (e,A) whose equilibria are constrained Pareto suboptimal $\mathscr{E}^{FRS} \in \mathscr{E}^{FR} \times \mathcal{A}$ is open and has full Lebesgue measure. In other words, for almost all regular economies with full trade $e \in \mathcal{E}^{RF}$, for any of its equilibrium $\xi = (x,\lambda,p)$, there exists an equilibrium with taxes $(\xi',t,\tau) \in \Xi \times \mathbb{R}^{L-1} \times \mathbb{R}$ such that x' Pareto dominates x.*

Proof. The aim is to show that 0 is a regular value of G, to then apply the Transversality theorem.

$$D_{\xi,\Delta,A}G(\xi,\Delta,e,A) = \begin{bmatrix} D_\xi F & D_\Delta F & D_A F \\ D_\xi \Phi & D_\Delta \Phi & D_A \Phi \end{bmatrix} = \begin{bmatrix} D_\xi F & 0 & 0 \\ D_\xi \Phi & D_\Delta \Phi & D_A \Phi \end{bmatrix}$$

$D_\Delta F = 0$ since F does not depend on Δ. When evaluated at equilibrium ξ^m, the perturbations of utility functions by A do not affect the equilibrium conditions $\big(D_{x_k}\tilde{u}_h(x_h^m, x_{-h}^m, A_h, A_{-h}) = D_{x_k}u_h(x_h^m, x_{-h}^m)$ for every $k \in H\big)$ therefore $D_A F(\xi^m) = 0$.

We know that for any regular economy, $D_\xi F$ has full rank. It thus remains to show that the matrix $[D_\Delta \Phi \; D_A \Phi]$ has full row rank.

Let's now consider the matrix $[D_\Delta \Phi \; D_A \Phi]$. From now on, without loss of generality, it is assumed that assumption 3 (policy: more tools than policy objectives) holds tightly; i.e. $L-1 = H$. Proving this case readily proves the less stringent cases in which $L-1 > H$ since the policy maker can always resort to taxing only H commodities. The advantage of focusing on the most stringent case $L-1 = H$ is that the matrix $\Phi(\xi,\Delta)$ is now square; there are no redundant columns to eliminate.

Case 1: $\delta_x^h \neq 0$, *for at least one* h.

Claim 1: $N(\delta_x)$ has full row rank.

As previously noted, if $\delta_{x_h} \neq 0$ for at least two $h \in H$, the matrix $N(\delta_x)$ has full row rank HL.

Claim 2: $(\delta_u, \delta_p^\backslash, \delta_\lambda) \neq (0,0,0)$.

By contradiction, assume $(\delta_u, \delta_p^\backslash, \delta_\lambda) = (0,0,0)$. The following equations are from $\Phi_{e,A}(\xi,\Delta) = 0$:

$\forall k, \qquad \sum_h \delta_{x_h} \; D^2_{x_k,x_h} u^h = 0$

$\forall h, \qquad \delta_{x_h} \; p = 0$

$\qquad \sum_h \delta_{x_h^\backslash} \; \lambda_h = 0$

Define $v_h = \delta_{x_h} \; \lambda_h$. For all h, δ_{x_h}, and thus v_h is orthogonal to p and at equilibrium ($F_{(e,u)}(\xi) = 0$, for every h), $D_{x_h}u^h(x_h, x_{-h}) - \lambda_h \; p = 0$, so p is collinear to $D_{x_h}u^h(x_h^*, x_{-h}^*)$.

From (14), $\delta_{x_h^C} = -\delta_{x_h^\backslash} \; p^\backslash$ therefore $\sum_h \lambda_h \delta_{x_h^C} = -p^\backslash \sum_h \lambda_h \delta_{x_h^\backslash} = 0$

Therefore, from (15), $\sum_h v^h = 0$

Post-multiplying (13) by v_k and summing over k, (13) becomes:

$\sum_k \sum_h \delta_{x_h} \; D^2_{x_k,x_h} u^h \; v_k = 0$;

after interchanging the order of (finite) summations it reads as follows:

$\sum_h \frac{v_h}{\lambda_h} \sum_k \; D^2_{x_k,x_h} u^h \; v_k = 0.$

$D_{x_h}u^h(x_h^*, x_{-h}^*) \; \delta_{x_h} = 0$, therefore, $D_{x_h}u^h(x_h^*, x_{-h}^*) \; v_h = 0$ Since $\delta_x^h \neq 0$, $v \neq 0$. Furthermore $\sum_h v^h = 0$ and $D_{x_h} u^h(x^*_h, x^*_{-h})\ v_h = 0$. Thus (13) contradicts assumption 3 (dominance of own effects).

Claim 3: After grouping lines and columns, the following matrix has full row rank:

$$[D_\Delta \Phi \quad D_u \Phi] = \begin{bmatrix} Du & D^2u & \Psi_t & \frac{I_{L-1}}{0} & 0 & N(\delta_x) \\ 0 & \Psi^T & 0 & 0 & 0 & 0 \\ 0 & -\Lambda & Z^\backslash & 0 & 0 & 0 \\ 0 & *_\lambda & *_{x^\backslash - e^\backslash} & 0 & *_{z^\backslash} & 0 \\ 0 & 0 & -1 & 0 & -H & 0 \\ \delta_u & \delta_x & \delta_\lambda & \delta_{p^\backslash} & \delta_\tau & 0 \end{bmatrix}.$$

The proof strategy relies on the perturbation method as presented in Villanacci et al (2002). $N(\delta_x)$ has full row rank by claim 1 and the configuration of the matrix ensures that the first row block can be perturbed independently of the other rows. By claim 2, the last line of the matrix can be perturbed independently of the other rows if the following $H + L \times HL + 1$ matrix extracted from the middle block

$$[D_\Delta \Phi \quad D_A \Phi] = \begin{bmatrix} \Psi_t^T & 0 \\ -\Lambda & 0 \\ *_\lambda & *_{z^\backslash} \\ 0 & -H \end{bmatrix}$$

has full row rank. The matrix of stacked matrices Ψ^T, $-\Lambda$ and $*_\lambda$ can readily be to have full row rank. Heuristically, the reasons for linear independence are that equilibrium prices of the block diagonal matrix Ψ are strictly positive (as a consequence of Assumption 1 ii)) whereas $-\Lambda$ is constituted of a row of H block diagonal matrices $I_{L-1}|0$ with a column of zeros stacked at the end. The rows of the matrix $*_{\lambda_h}^T \ = -D_t\ [\lambda_h\ \frac{t_h}{0}]$ are linearly independent from the others since every individual trades every commodities and $L - 1 \geq H$ by assumption 3, thus t_h is strictly positive for at least one consumer and strictly negative for at least one other consumer. This contrast with the diagonal patterns of the upper rows. Finally, since $H \neq 0$ the last row can be perturbed and so independently of the others.

Case 2: $\delta_{x_h} = 0$, for all h.

$\Phi_{e,u}(\xi, \Delta) = 0$ requires $\sum_h \delta_{u^h} D_{x_k} u^h + \sum_h \delta_{x_h} D^2_{x_h, x_k} u^h + \delta_{\lambda_k} p + \delta_{p^\backslash} I_{L-1}|0 = 0$ which, with $\delta_{x_h} = 0$ for all h, becomes:

$\sum_h \delta_{u^h} D_{x_k} u^h + \delta_{\lambda_k} p + \delta_{p^\backslash} I_{L-1}|0 = 0$

Since at equilibrium, $D_{x_h}u^h(x_h, x_{-h}) - \lambda_h p = 0$, equivalently,
$\sum_h \delta_{u^h} D_{x_k}u^h + \delta_{\lambda_k} D_{x_k}u^h(x_k, x_{-k}) + \delta_{p\setminus} I_{L-1}|0 = 0$
This contradicts assumption 4. Case 2 does not happen.
Case 1 and Case 2 cover all cases. Since Case 2 cannot happen, Claim 3 of Case 1 guarantees that 0 is a regular value of G. It only remains to apply the Transversality theorem and the Regular Value theorem.
$G : \Xi \times \mathcal{S}^{H+\dim\Xi} \times \mathbb{R}^{HL}_{++} \times \mathcal{A} \rightarrow \mathbb{R}^{\dim\Xi} \times \mathbb{R}^H \times \mathbb{R}^{\dim\Xi} \times \mathbb{R}$ is a smooth map of manifolds.
0 is a regular value of G; the Transversality theorem guarantees that there exists a set $\mathscr{E}^{FRS} \subset \mathscr{E}^{FR} \times \mathcal{A}$ that has full Lebesgue measure, and such that:

for every $(e, A) \in \mathscr{E}^{FRS}$, 0 is a regular value of $G_{e,A} : \Xi \times \mathcal{S}^{H+\dim\Xi} \rightarrow \mathbb{R}^{\dim\Xi} \times \mathbb{R}^H \times \mathbb{R}^{\dim\Xi} \times \mathbb{R}$.

For every $(e, A) \in \mathscr{E}^{FRS}$, $G^{-1}_{e,A}(0) = \emptyset$; otherwise, by the Regular Value theorem, $G^{-1}_{e,A}(0)$ would be regular submanifold of negative dimension $0 < (\dim\Xi + H + \dim\Xi) - (\dim\Xi + H + \dim\Xi + 1)$, a mathematical impossibility. However, existence of equilibrium for economies satisfying assumption 1 guarantees that $F^{-1}_{e,A}(0) \neq \emptyset$. It must thus be that $\Phi^{-1}_{e,A}(0) = \emptyset$. Equilibria of $(e, A) \in \mathscr{E}^{FRS}$ are constrained Pareto suboptimal. □

5 Conclusion

The analysis of a model of a market with external effects allowed us to identify a condition that ensures that, generically, "good tax packages" exist. This condition (Assumption 4) requires marginal externalities to be diverse at equilibrium. It sheds light on the mechanism by which taxes generate a Pareto improvement. Since non-separable externalities may have an effect on consumer demands, our result raises the following question: can these welfare-enhancing tax packages be identified from market data?

6 Appendix A: Computations

Remark: results using assumption 4 are not vacuously true (continued).

For simplicity, $EXT(\alpha) =$

$$\begin{bmatrix} I_{L-1} & \dots & I_{L-1} & 0 \\ D_{x_1^\setminus} u^1 & \dots & D_{x_H^\setminus} u^1 & D_{x^C} u^H \\ \vdots & \ddots & \vdots & \vdots \\ D_{x_1^\setminus} u^H & \dots & D_{x_H^\setminus} u^H & D_{x^C} u^H \\ \alpha_1 \, D_{x_1^\setminus} u^1 & \dots & \alpha_H \, D_{x_H^\setminus} u^H & \alpha \otimes D_{x^C} u^H \end{bmatrix} =: \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ \alpha & \beta & \epsilon & p & r & s & \mu & 0 \\ a & b & e & \pi & \rho & \sigma & 0 & \nu \\ \alpha_1\alpha & \alpha_1\beta & \alpha_1\epsilon & \alpha_2\pi & \alpha_2\rho & \alpha_2\sigma & \alpha_1\mu & \alpha_2\rho \end{bmatrix}$$

After column operations

$$\text{Rank}\, EXT(\alpha_1, \alpha_2) = \text{Rank} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & p-\alpha & r-\beta & s-\epsilon & \mu & 0 \\ 0 & 0 & 0 & \pi - a & \rho - b & \sigma - e & 0 & \nu \\ 0 & 0 & 0 & \alpha_2\pi - \alpha_1\alpha & \alpha_2\rho - \alpha_1\beta & \alpha_2\sigma - \alpha_1\epsilon & \alpha_1\mu & \alpha_2\nu \end{bmatrix}$$

after column operations

$$= \text{Rank} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \nu \\ 0 & 0 & 0 & * & ** & *** & 0 & 0 \end{bmatrix}$$

where
$* := \alpha_2\pi - \alpha_1\alpha - \alpha_1(p - \alpha) - \alpha_2(\pi - a) = \alpha_2 a - \alpha_1 p$
$** := \alpha_2\rho - \alpha_1\beta - \alpha_1(r - \beta) - \alpha_2(\rho - b) = \alpha_2 b - \alpha_1 r$
$*** := \alpha_2\sigma - \alpha_1\epsilon - \alpha_1(s - \epsilon) - \alpha_2(\sigma - e) = \alpha_2 e - \alpha_1 s$
Assumption 4 requires that EXT has full row rank. In this simple case with no externalities from consumption of the numéraire, it requires that there is no $(\alpha_1, \alpha_2) \neq (0, 0)$ such that:

$$\alpha_2 a - \alpha_1 p = \alpha_2 b - \alpha_1 r = \alpha_2 e - \alpha_1 s$$

which, with the initial notation, corresponds to:

$$\alpha_2 \frac{\partial u_2}{\partial x_1^1} - \alpha_1 \frac{\partial u_1}{\partial x_2^1} = \alpha_2 \frac{\partial u_2}{\partial x_1^2} - \alpha_1 \frac{\partial u_1}{\partial x_2^2} = \alpha_2 \frac{\partial u_2}{\partial x_1^3} - \alpha_1 \frac{\partial u_1}{\partial x_2^3}.$$

This condition can be rewritten as follows:

$$\frac{\frac{\partial u_1}{\partial x_2^2} - \frac{\partial u_1}{\partial x_2^3}}{\frac{\partial u_1}{\partial x_2^2} - \frac{\partial u_1}{\partial x_2^1}} = \frac{\frac{\partial u_2}{\partial x_1^2} - \frac{\partial u_2}{\partial x_1^3}}{\frac{\partial u_2}{\partial x_1^2} - \frac{\partial u_2}{\partial x_1^1}}.$$

7 Appendix B: Notation and conventions

- Matrices are always evaluated at an equilibrium allocation unless specified otherwise.

- The word perturbation is used as it is used in Villanacci et al (2002) pp21-27 for the computation of the rank of matrices. It is also used to refer to the variables with which derivative is taken. For example: 'endowments are perturbed to perturb the last row of the matrix'. The last row is perturbed for computation of the rank and the variable with respect to which derivative is taken is the endowment.

- Euclidean spaces are considered as manifolds endowed with the standard smooth structure.

- $\{1, ..., h, ..., H\}$ is the set of individuals.

- $\{1, ..., l, ..., L\}$ is the set of commodities.

- p denote commodity prices.

- Individual choice variables: x denotes individual consumption; z denote excess demand.

- x_h^l is the consumption of commodity l by individual h.

- $\sum_h$ is shorthand notation for $\sum_{h \in H}$. In many occasions, when the index clearly identifies the set over which it is summed, the shorthand notation is used.

- $\lambda_L H$ denotes the Lebesgue measure on $\mathbb{R}_{++}^{LH}$, while λ_h denotes the Lagrange multiplier from the constrained optimization of the consumer problem.

- $\frac{t_h}{0}$ always refers to two matrices stacked over one another. Similarly, $I_{L-1}|0$ denotes the identity matrix of size $L-1$ with a column of zero stacked at the end.

- when a function ischange of domain when defining $F := F_e$

- $\Delta = (\delta_u, \delta_x, \delta_\lambda, \delta_{p\backslash}, \delta_\tau)$, an element of $\mathcal{S}^{H+\dim\Xi}$ The index only indicates the dimension of the element of Δ. That is, for example $\delta_u \in \mathbb{R}^H$, $\delta_x \in \mathbb{R}^{LH}$.

- Let $i \neq h$. In a neighborhood of equilibrium m that is contained in the support of b_m:

$$D_{x_h}\tilde{u}_h(x_h, x_{-h}, A_h, A_{-h}) = D_{x_h}u_h(x_h, x_{-h}) + A_h\ (x_h - x_h^m) + A_i\ (x_i - x_i^m)$$

$$D_{x_h}\tilde{u}_h(x_h^m, x_{-h}^m, A_h, A_{-h}) = D_{x_h}u_h(x_h^m, x_{-h}^m)$$

$$D_{x_i}\tilde{u}_h(x_h, x_{-h}, A_h, A_{-h}) = D_{x_i}u_h(x_h, x_{-h}) + \sum_m b_m(x^m)[(x_h - x_h^m)\ A_i]$$

$$D_{x_i}\tilde{u}_h(x_h^m, x_{-h}^m, A_h, A_{-h}) = D_{x_i}u_h(x_h^m, x_{-h}^m)$$

$$D_{x_k}\tilde{u}_h(x_h^m, x_{-h}^m, A_h, A_{-h}) = D_{x_k}u_h(x_h^m, x_{-h}^m)$$

$$D^2_{x_h,x_h}\tilde{u}_h(x_h, x_{-h}, A_h, A_{-h}) = D^2_{x_h,x_h}u_h(x_h, x_{-h}) + A_h$$

$$D^2_{x_h,x_i}\tilde{u}_h(x_h, x_{-h}, A_h, A_{-h}) = D^2_{x_h,x_i}u_h(x_h, x_{-h}) + A_h + A_i$$

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