

Regional Trade Agreements with Global Value Chains

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Abstract

This paper uses data on directional trade flows and Regional Trade Agreements (RTA) to i) estimate the effects of RTA on trade flows and ii) assess the importance of Global Value Chains for these effects. Based on a Difference-in-Difference identification strategy, we find that RTAs are associated with: (1) an increase in trade within the region, (2) a decrease in inflows to the region, and (3) an increase in outflows from the region. The first two findings can be understood as trade creation and trade diversion due to a shift in demand associated with the lower trade barriers within the region and Rules of Origin as an implicit trade barrier for imports from the rest of the world. Global Value Chains are most relevant to understand the third finding. The key determinant of the increase in outflows is the importance of the *Regional* Value Chains for *imports of intermediates* by members of the region. We then propose an augmented version of a standard intra-industry trade model à la Melitz (2003) with input-output linkages. Our model with input trade is able to qualitatively account for all three empirical findings, while a standard model of trade in final goods cannot replicate the increase in outflows from the agreement region. We use it to revisit the debate on whether regionalism begets more regionalism or multilateralism. We find that the incentive for the region is towards multilateralism whereas the incentive for the rest of the world is towards regionalism.

Keywords: Trade agreements, trade diversion, trade policy, global value chains.

JEL classification: F1, F14, F15, F42, F60.

1 Introduction

The rise of Global Value Chains (GVCs) is a dominating feature of the recent evolution in the structure of international trade. In the OECD, the import content of exports increased by 63% between 1995 and 2011, reaching a value of 24.3% on average.¹ This internationalization of production through global value chains calls for a reevaluation of the effects of regional trade agreements (RTA) on trade flows and of the theory on regionalism.

Since [Viner \(1950\)](#), the effects of regional trade agreements are framed in terms of two concepts: trade creation and trade diversion. An increase in intra-regional trade associated with a RTA is called *trade creation*. In turn, a decrease in trade from the rest of the world to the region is called *trade diversion*. Trade creation is thought to be associated with resources being shifted from relatively inefficient domestic suppliers towards more efficient regional suppliers. In contrast, trade diversion could be the result of resources being shifted from efficient extra-regional suppliers towards inefficient regional suppliers. The balance between trade creation and trade diversion emerges as a key consideration for the desirability of a regional trade agreement.

The emergence of a consensus that globalization improves industry performance² motivates our reevaluation, both empirically and theoretically, of regional trade agreements and their effects. First, we study empirically the evolution of trade associated with the implementation of regional trade agreements. Second, we propose a model of intra-industry trade with GVCs in which globalization improves industry performance. We find that our model is consistent with our empirical findings in ways that standard trade models are not. Third, our model sheds new light on the determinants of the desirability of regional trade agreements.³

First, this paper provides estimates of the effects of regional trade agreements on trade within the region, and trade between the region and the rest of the world. Our identification strategy exploits the panel structure of the data to control for country-pair, and country-year fixed effects. Importantly, directed trade flows allows to separately identify the extent of trade agreements for inflows to, and for outflows from, the region. The separate identification of the effects of trade agreements on inflows to, and outflows from, the region for a large sample of countries and trade agreements is the first empirical contribution of this paper.

We find evidence of an increase in intra-regional trade, which is in line with the existing literature. We also find strong evidence of a decrease in inflows to the region and an increase in outflows from the region. The evidence of strong effects on inflows and outflows is, to the best of our knowledge, new to the literature. To be sure, our results on inflows and outflows

¹For some countries such as Luxembourg or Belgium, imports make up more than a third of their exports.

²See [De Loecker and Goldberg \(2014\)](#).

³This part is still in progress.

are also in line with results previously found in the literature: there is a small *net* effect on trade between the region and the rest of the world. However, looking at inflows and outflows separately uncovers the strong opposite effects on inflows and outflows underlying the small net effect on total flows. Our findings on the effects of RTA on intra-regional flows and on inflows to the region are suggestive of trade creation and trade diversion; two mechanisms that have been studied extensively. Less studied is our finding of an increase in outflows from the region associated with RTAs.

We then refine our analysis to account for participation in Global Value Chains (GVC). In particular, we construct two different indices of participation to the GVC. The first index captures the importance of the country as a supplier of inputs to the GVC. The second index measures the importance of the regional part of the GVC for a given country. The explicit account of GVC for the study of the effect of regional trade agreements on trade flows is the second empirical contribution of this paper.

We find that accounting for Global Value Chains matters most for the increase in outflows from the regions associated with Regional Trade Agreements. In particular, our index measuring the importance of the regional part of the GVC accounts for the positive effect of RTAs on outflows from the region. We interpret these findings as a manifestation that globalization benefits industry performance through regional input-output networks.

Second, we extend a model of intra-industry trade with firm heterogeneity à la [Chaney \(2008\)](#) with GVCs. In our setup, firms use labor and intermediate inputs to produce goods. Firms benefit from having access to a broad set of intermediate inputs, just like consumers have a preference for a broad set of varieties. Firms' export technology exhibits increasing returns to scale. Firms decide whether to export to other countries, which endogenously determines the global network of value chains. Importantly, the marginal cost of production depends on a firm's access to intermediate inputs, which is determined endogenously by the global network of value chains. Each firm's marginal cost is endogenously determined as a result of all other firms' export decisions. Numerical simulations show that our model generate an increase in outflows from the region. Standard models of trade in final goods fail to account for this empirical finding.

In a context of production chains that are increasingly global, our findings have implications for the debate on regionalism and multilateralism. We use the literature on the determinants of optimal trade policy to interpret our findings' implications for this debate. Our findings suggest that the effect of RTAs on trade volumes lowers the region's incentive to have trade barriers on inflows, while it increases the incentives for the rest of the world to raise trade barriers on outflows from the region.

Literature.

Estimating the effects of regional trade agreements requires the building of a counterfactual world. We use gravity equations to predict the counterfactual as in [Carrere \(2006\)](#), [Magee \(2008\)](#), [Baier and Bergstrand \(2007\)](#) and [Baier and Bergstrand \(2015\)](#).

Our empirical approach is most closely related to [Magee \(2008\)](#), which estimates the effects of trade agreement using a panel of 133 countries from 1980 to 1998. Also closely related is [Carrere \(2006\)](#) who uses a gravity model to assess trade creation and trade diversion effects. Our contribution comes from the broader coverage of Regional Trade Agreements and countries, our focus on the effect of RTA on inflows to and outflows from the region, and on our account of the role of Global Value Chains.

Also related is [Baier and Bergstrand \(2007\)](#) and [Baier and Bergstrand \(2015\)](#) who address the problem of endogeneity related to free trade agreements and trade flows by using panel data and average treatment effects (ATEs). They find positive estimates and conclude that free trade agreements increase members' international trade.

[Anderson and Yotov \(2016\)](#) refer to a gravity model to estimate the effects of trade agreements on terms of trade and global efficiency. They face two main problems: heteroskedasticity in trade flows data, and endogeneity due to the two way causality. To address the first one, they use the Poisson pseudo maximum likelihood (PPML). To address the second one, they introduce two variables, one for trade agreements between countries with low most-favored-nation tariffs (MFN), the other for trade agreements between countries with high MFN tariffs. They find an increase in the global efficiency of manufactures trade over the period 1990-2002.

The behavior of trade flows following a regional trade agreement is also impacted by the depth of the agreement, as analyzed in [Mattoo et al. \(2019\)](#). Using a sample of 96 countries for the period 2002-2014, they show that deep agreements lead to more trade creation and less trade diversion than shallow agreements.

Additionally, beyond complementarity in trade policy, GVCs also create strong interdependence of GDP fluctuation at business cycle frequency and are associated with network propagation effects of any reform aiming at reducing the cost of cross-country trade.⁴ Moreover, the segmentation of production across countries also has significant implications for the magnitude of estimated trade elasticities ([Amiti et al. \(2014\)](#), [de Soyres et al. \(2018\)](#)).

The model we propose follows [de Soyres and Gaillard \(2019\)](#) in modelling GVC in a model of intra-industry trade with firm heterogeneity à la [Chaney \(2008\)](#) and [Melitz \(2003\)](#). Importantly, firms are engaged in both importing and exporting activities, creating a link

⁴See [Gunnella et al. \(2019\)](#) and [de Soyres and Gaillard \(2019\)](#) for the former point, [Boehm et al. \(2019\)](#) and [de Soyres et al. \(2020\)](#) for the latter.

between access to cheaper inputs from abroad and export prices to other countries. Following [Yi \(2003\)](#), models of GVC have emphasized multi-stage production. In much of the literature, firms make a discrete choice of location for each stage of production. This is a challenging problem to solve; recent advances are [Antràs et al. \(2017\)](#), [Antràs and de Gortari \(2020\)](#) and [de Gortari \(2020\)](#). Our model features endogenous GVCs which matter for each firms' cost function. Our focus on the *aggregate implications of GVCs* allows us to bypass the challenge posed by a discrete choice of location for each stage of production. As a result, our model retains much of the tractability of models of trade in final goods.

Finally, [Estevadeordal et al. \(2008\)](#) study the impact of regional trade agreements on trade liberalization towards non-members. Focusing on Latin American countries, they found “complementary effects” in the sense that a preferential tariff reduction leads to a reduction in the external tariff. [Freund and Ornelas \(2010\)](#) provide an insightful review of the literature on regionalism. We follow [Frankel \(1997\)](#) in defining trade creation as an increase in trade between members of a preferential trade region and trade diversion as a decrease in trade between members of a preferential trade region and the rest of the world. The empirical findings of this paper, combined with theories of optimal trade policy suggest two opposing forces towards greater global integration: the response of trade flows to an RTA gives members an incentive to reduce trade barriers towards non-members; however it gives non-members an incentive to increase trade barriers towards the region. The incentive for members to lower trade barriers echoes the literature on multilateralism ([Estevadeordal et al. \(2008\)](#), [Bagwell and Staiger \(1999\)](#)).

The rest of the paper is organized as follows. Section 2 presents the data sources and the construction of key variables. Section 3 estimates the effects of Regional Trade Agreements on trade flows. Section 4 presents a simple model with GVC which is shown to be qualitatively consistent with our empirical findings. Section 5 discusses the implications of our findings for the debate on regionalism and multilateralism.

2 Data

2.1 Data on Trade Flows

We collect data on bilateral trade flows from the Observatory of Economic Complexity (MIT). This database covers 215 countries over the period 1962-2014. The data are classified according the 4-digit Standard International Trade Classification (SITC), Revision 2. Only products and commodities are considered. Then, we aggregate these bilateral trade flows at the country-

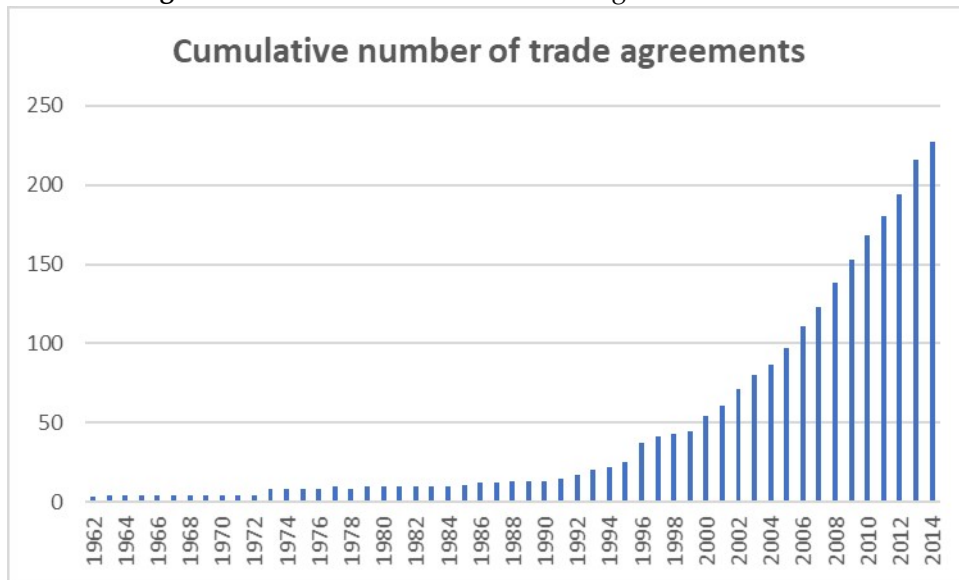
level.

To classify trade flows into final and intermediate goods, we use a concordance table⁵ from SITC Rev. 2 to Broad Economic Categories (BEC). We then classify goods into five categories: primary, semi-finished goods, parts and components, capital goods, consumption goods, and a residual category called goods non-specified. We group these categories into intermediate and final goods as follows: intermediate goods are primary goods, semi-finished goods, parts and components; and final goods are consumption goods and capital goods.

2.2 Data on Trade Agreements

The data on trade agreements comes from a World Bank database⁶. We use the “bilateral observations” file where each observation is a country-pair agreement for one year. There are 189 countries. The period covered is 1958-2015, during which 279 agreements were signed. We delete trade agreements on services only, which leaves us with 227 trade agreements in our final database. Figure 1 shows the cumulative number of trade agreements over time. We observe a net increase in the number of trade agreements since 1990.

Figure 1. Cumulative number of trade agreement over time



Based on the World Bank database on trade agreements, see [Hofmann Claudia and Ruta \(2017\)](#).

⁵The concordance table from SITC Rev2 to BEC can be found on the UN Trade Statistics webpage: <https://unstats.un.org/unsd/trade/classifications/correspondence-tables.asp> .

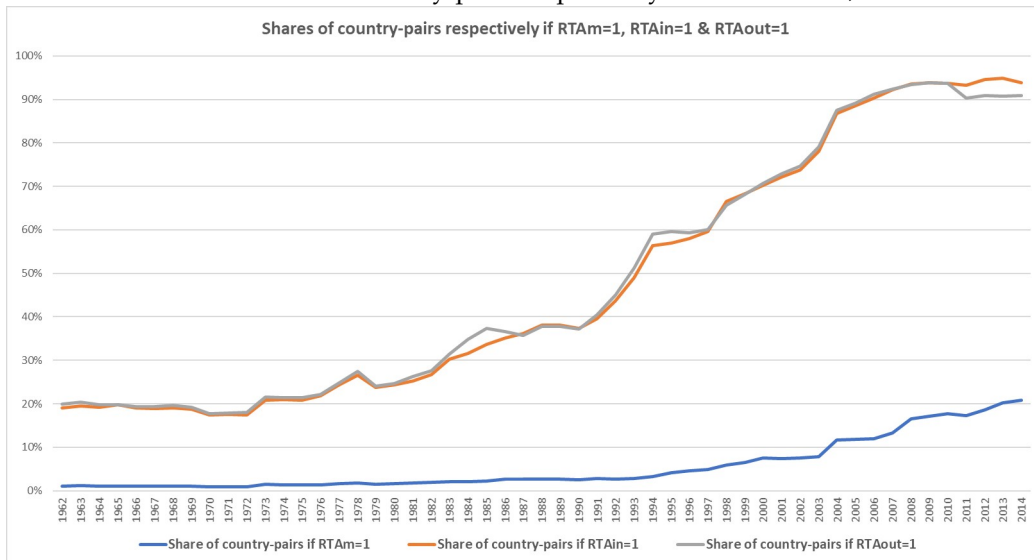
⁶See [Hofmann Claudia and Ruta \(2017\)](#)

2.3 Variables

We create three variables encoding countries' membership in the different trade agreements. The first variable, denoted RTA_m , is equal to 1 when the country of "origin" and the country of "destination" are both members of the same trade agreement; RTA_m is equal to 0 otherwise. We use the directed trade flows to create variables encoding inflows and outflows from the region. That is, the second variable, denoted RTA_{in} , is equal to 1 when the country of "origin" is not a member of any trade agreements to which the country of "destination" belongs; otherwise RTA_{in} is equal to 0. The third variable, denoted RTA_{out} , is equal to 1 when the country of "destination" is not a member of any trade agreement to which the country of "origin" belongs; otherwise, RTA_{out} is equal to 0. Trade agreements are complex and vary in their coverage and depth. Our analysis of more than two hundred trade agreements limits us in the extent to which we can code the complexity of preferential trade agreements. Our binary coding of Regional Trade Agreements provides a coarse measure of the evolution of preferential trade regions over time, which risks being a source of attenuation bias. As we will see, we still find strong effects.

Figure 2 displays the evolution of the shares of country-pairs that are part of a regional trade agreement ($RTA_m = 1$), an exporter to a regional trading block ($RTA_{in} = 1$), and an importer from a regional trading block ($RTA_{out} = 1$). We see that over the past fifty years, the share of countries that are part of a regional trade agreement rose from 2% to over 20%. The share of countries that do not trade with any regional trade block decreased from 80% to below 10%.

Figure 2. Evolution of the shares of country-pairs respectively when $RTA_m=1$, $RTA_{in}=1$ & $RTA_{out}=1$



Participation in Global Value Chains is multifaceted. A country's participation ought to be differentiated by import and export status. The analysis of Regional Trade Agreements suggests decomposing Global Value Chains into their regional components. We construct two variables to capture the degree of participation of a country in Global Value Chains. The first such variable, denoted *bilateral*, is the share of intermediate goods that the country of "origin" exports to the country of "destination". The variable *bilateral* accounts for a country's importance in Global Value Chains as an exporter. The second variable, denoted *bregional*, is the share of intermediate inputs that the country of "origin" is importing from other countries that are part of the same regional trade agreement. The variable *regional* accounts for the importance of the regional component of GVC to the imports of a given country.

As we will discuss in the next section on our empirical approach, trade policy is endogenous. Countries that are natural trade partners may be more inclined to sign a trade agreement (Krugman (1991)). Our data provides empirical support to this hypothesis. On average, upon signing an agreement, a country-pair that just signed an agreement trades roughly twice as much as a country-pair where only one country only joined the trade agreement. More precisely: for each country and agreement, we normalized to 100 the trade flows between two countries signing a trade agreement, on the year the agreement is put into force. We can then compare this number (for the same year) to the trade flows between a member country to a non-member country. On average across all countries and trade agreements, we find that trade with non members is only 55.94% of trade between members, on the first year of the agreement.⁷

3 Regional trade agreements and trade flows

3.1 Setup

Difference-in-differences is the natural identification strategy for a panel setting in which a policy affects groups. Our identification of the effects of trade agreements comes from comparing trade flows, for the relevant country-pairs, before and after a trade agreement comes into effect, for the 227 trade agreements that were signed between 1958 and 2015. Importantly, our specification controls for country-pair and country-time fixed effects.⁸ The

⁷In order to avoid outliers, we keep 92.5% of the observations.

⁸The directional trade data we use is rich enough that we could expand on the country-time fixed effects to control for country-time by exporter and importer status. Country-time fixed effects by exporter and importer status would however absorb all the variations that we use to separately identify the effects of trade agreements on inflows and outflows.

country pair fixed effects not only accounts for standard time invariant covariates of gravity models such as geography, institutions, and culture, but also for unobserved time invariant heterogeneity. Country-time fixed effects controls for all country-specific factors in a given year; these include observable covariates such as GDP, domestic politics such as election year, weather and all other country-specific unobservables. We estimate the following regressions by Ordinary Least Squares:

$$\ln(\text{Trade}_{ijt}) = \delta_m \text{RTAm}_{ijt} + \delta_{in} \text{RTAin}_{ijt} + \delta_{out} \text{RTAout}_{ijt} + \beta_{ij} + \beta_{it} + \beta_{jt} + \epsilon_{ijt} . \quad (1)$$

A positive coefficient δ_m suggests evidence of trade creation where members respond to a decrease in trade barriers by trading more. A negative coefficient δ_{in} would suggest trade diversion where members of the newly created trade block import less from the rest of the world. A positive coefficient δ_{out} would indicate that members of the newly created trade block export more to the rest of the world. The interpretation of the coefficients is sensitive to endogeneity issues to which we now turn.

Endogeneity of trade agreements: While our baseline specification aims to capture the effect of trade agreements on trade flows, the causality may play in the other direction: countries self-select into membership, importer, and exporter to the trade block. In other words, members of the trade agreement may be a selected group of countries inclined to trading with one another. This concern is partly addressed by country-pair fixed effects. Country-pair fixed effects control for the extent to which self-selection into membership is based on an inclination to a high level of trading. The extent to which self-selection into membership is based on an inclination to a high increase in trade over time, however, is not controlled for. Such self-selection into trade blocks due to an inclination to a high increase in trade over time would lead to over-estimate trade creation; that is the estimator for δ_m would be biased upward. Likewise, for trade partners which were not selected to be part of the agreement, the results would over-estimate trade diversion; that is δ_{in} and δ_{out} would be biased downwards.

Trade agreements vary in their coverage and depth for trade within the region. They do also affect the region's trade policy with the rest of the world. [Estevadeordal et al. \(2008\)](#) find that preferential tariff reduction leads to a reduction of the tariff applied to non-members. In contrast, Rules of Origin requirements would tend to have the opposite effect. Our binary coding of participation in various trade agreements provides a coarse measure of the change in trade barriers between countries. Such measurement error is a source of attenuation bias which would tend to compensate for some of the biases due to potential reverse causality.

Clustering standard errors: An observation is a unidirectional trade flow for a country-pair in a given year. Standard errors are robust to clustering at the country-pair level, which

accounts for correlation across either directions in the bilateral relationship as well as for serial correlation across time. That is, we allow for the error term to have a fixed country-pair component common to both pairs (ij) and (ji) and for serial correlation within bilateral country pairs.

3.2 Results

Table 1 presents the OLS estimates of the within-group transformation of the regression equation (1). The coefficient for RTA_m is positive and highly significant. When two countries sign a trade agreement, trade flows between them increase by an estimated 27%. The coefficient for RTA_{in} is -0.22. When a country becomes a member of a trade agreement, the imports coming from a country outside the trade agreement decrease by an estimated 22%. The coefficient for RTA_{out} is 0.296: when a country signs a trade agreement, its exports increase by an estimated 30% to non-member countries.

	log(trade)
RTAm	0.274*** (9.99)
RTAin	-0.220*** (-7.44)
RTAout	0.296*** (9.80)
Country pair FE	Yes
Country \times time FE	Yes
Observations	796,107
R ²	0.7945

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 1. OLS estimates of regression equation (1)

As expected, two countries entering a trade agreement is associated with an increase in trade between them. This is evidence of trade creation, which corroborates what was found in previous studies. More interesting is the associated strong decrease in inflows to, and a strong increase in outflows from, the region. While this does not contradict previous findings of a small effect on total trade with the rest of the world, our findings uncover strong underlying effects on inflows and outflows.

The decrease in inflows to the region following the implementation of a trade agreement is evidence of trade diversion. The lower trade barriers within the region make imports from the

region relatively more appealing. As a result, members source more from within the region and less from outside the region. While regional trade agreements lower trade barriers within the region, their effect on trade barriers with the rest of the world is ambiguous. On the one hand, trade agreements feature Rules of Origin clauses which regulate the origin of the goods benefiting from lower trade barriers. Rules of Origin are a form of trade barrier on inflow to the region which is an additional factor explaining the decrease in inflows to the region associated with a regional trade agreement. On the other hand, [Estevadeordal et al. \(2008\)](#) find evidence that regional trade agreements lead to lower external trade barriers. Overall, our findings suggest that, despite complementarities in tariff reductions within the region and on imports, regional trade agreements tend to lower imports from the rest of world.

The strong increase in outflows from the region associated with a regional trade agreements is less expected. Trade diversion lowers the demand for imports from the rest of the world which could have adverse effects on production abroad thereby promoting exports from the region. The emergence of a consensus that “globalization improves industry performance” (cf. [De Loecker and Goldberg \(2014\)](#)) suggests a complementary mechanism contributing to the observed increase in outflows. Industries within the region benefit from greater access to their regional part of the Global Value Chains. The associated increase in performance fosters exports from the region. The next section investigates the extent to which Global Value Chains contribute to our findings.

The importance of separating inflows from outflows.

It is interesting to contrast our findings to a setup similar to [Magee \(2008\)](#), where the effect of RTA on total flows, an aggregate of inflows and outflows, is measured. To do so, we define a dummy variable $RTA_{in-or-out}$ which is equal to one if and only if either RTA_{in} or RTA_{out} is equal to one. Such a dummy captures the changes in total trade flows between a member of a RTA and a non-member of a RTA compared to the control group composed of country-pairs that do not join a RTA the same time period. We estimate the role of RTAs based on the following equation:

$$\ln(Trade_{ijt}) = \delta_m RTA_{mijt} + \delta_{in-or-out} RTA_{in-or-out}_{ijt} + \beta_{ij} + \beta_{it} + \beta_{jt} + \epsilon_{ijt} . \quad (2)$$

Results presented in Table 2 show that if one does not account for inflows outflows separately, the net effect of RTAs on flows between a member and a non member is positive overall. This is the net effect of two strong underlying effects documented in Table 1.

	log(trade)
RTAm	0.280*** (10.27)
RTAin-or-out	0.103*** (4.56)
Country-Pair FE	Yes
Country \times time FE	Yes
Observations	796,107
R2	0.793

Table 2. OLS estimates of regression equation (2)

3.3 Accounting for Global Value Chains

To assess the contribution of Global Value Chains to the effects of regional trade agreements on trade flows, we augment the analysis from the previous section to include two measures of participation in Global Value Chains. As described in subsection 2.3, our first measure, denoted *bilateral*, is the share of intermediates in bilateral exports from the country of origin. The second measure, denoted *regional*, is the country of origin's share of intermediates in imports from the region. The variable *bilateral* captures the importance of a country as a supplier of intermediate goods to the Global Value Chains. The variable *regional* captures the importance of the region's part of the Global Value Chains in supplying intermediate goods to a given country. The augmented regression equation reads as follows:

$$\begin{aligned}
\log(\text{Trade}_{ijt}) = & \delta_m \text{RTAm}_{ijt} + \delta_{in} \text{RTAin}_{ijt} + \delta_{out} \text{RTAout}_{ijt} \\
& + \delta_1 \text{bilateral}_{ijt} + \delta_2 \text{regional}_{ijt} \\
& + \delta_{m1} \text{RTAm} \times \text{bilateral}_{ijt} + \delta_{m2} \text{RTAm} \times \text{regional}_{ijt} \\
& + \delta_{in1} \text{RTAin} \times \text{bilateral}_{ijt} + \delta_{in2} \text{RTAin} \times \text{regional}_{ijt} \\
& + \delta_{out1} \text{RTAout} \times \text{bilateral}_{ijt} + \delta_{out2} \text{RTAout} \times \text{regional}_{ijt} \\
& + \beta_{ij} + \beta_{it} + \beta_{jt} + \epsilon_{ijt}
\end{aligned} \tag{3}$$

Results accounting for GVC.

Table 3 presents the OLS estimates of the within-group transformation of the regression equation (3).

To understand the importance of Global Value Chains (GVC) for the effect of Regional Trade Agreements on trade flows, it is insightful to compare the results from model (1), which

	log(trade)
RTAm	0.215** (2.91)
RTAin	-0.329*** (-6.05)
RTAout	-0.097 (-1.57)
<i>bilateral</i>	0.898*** (33.92)
<i>regional</i>	-0.196 (-1.35)
RTAm \times <i>bilateral</i>	0.094 (1.55)
RTAm \times <i>regional</i>	-0.012 (-0.10)
RTAin \times <i>bilateral</i>	0.093** (3.00)
RTAin \times <i>regional</i>	-0.083* (-2.09)
RTAout \times <i>bilateral</i>	-0.706*** (-22.91)
RTAout \times <i>regional</i>	1.47*** (10.05)
Country pair FE	Yes
Country \times time FE	Yes
Observations	796,107
R2	0.7982

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3. OLS estimates of regression equation (3)

does not account for GVC, with the results from model (3), which does account for GVC. Comparing Table 1 to Table 3, one main message emerges: participation in GVC is a key determinant of the strong positive effect of RTAs on outflows from the regions. It does not, however, contribute much to the creation of trade within regions and the diversion of inflows to the region.

First, we look at the effect of RTAs on the creation of trade within the region. Comparing Table 1 to Table 3, the coefficients for RTA_m are of similar magnitude. Our two measures of participation to GVC do not explain much of the trade creation resulting from RTA. It is worth noting that participation in GVC does, however, contribute mildly to trade creation. Likewise, comparing coefficient for RTA_{in} gives a similar picture of small effects of GVC on trade diversion for inflows to the region.

Accounting for participation in GVC gives rich insights into the mechanisms explaining the positive effect of RTAs on outflows from the regions. In the absence of GVC, that is if

bilateral and *regional* were 0, the coefficient for RTA_{out} indicates that RTA would be associated with mild trade diversion of outflows from the region. Comparing Table 1 to Table 3, the drop in magnitude and the change of sign for coefficient RTA_{out} reveals that GVC play a major role in explaining the increase in outflows from the region.

Our two measures of participation in GVC allows to gain insights into the determinants of the increase in outflows from the regions. The effect of participation in exports to the Global Value Chains on outflows from the region is negative. Countries that mostly supply to the Global Value Chains are most affected by the shift in demand in the region. In contrast, the effect of participation in importing from the Regional Value Chains is positive and is the key determinant of the overall positive effect. Countries that rely most on the Regional Value Chain to source their inputs are the ones that benefit the most from the RTA. The boost in industry performance fosters more exports from the region to the rest of the world.

	if RTAm=1	if RTAin=1	if RTAout=1
Mean(<i>bilateral</i>)	0.33	0.42	0.41
Median(<i>bilateral</i>)	0.3	0.38	0.39
Mean(<i>regional</i>)	0.45	0.37	0.53
Median(<i>regional</i>)	0.52	0.48	0.56

Table 4. Mean and Median values of GVC variables, disentangled by RTA dummies

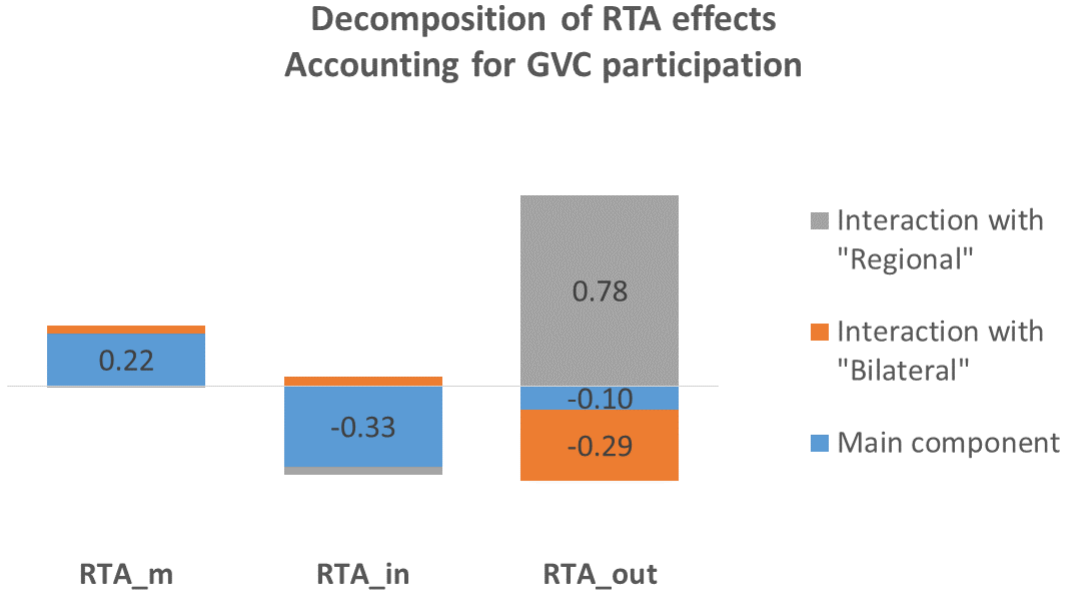
3.4 Timing of the effects of Regional Trade Agreements

The total effects of RTA on trade flows can be decomposed in effects at different points in time. We decompose the total effect on trade flows, as documented in Table 1, into anticipatory and lagged effects. The following estimating equation shows that we use five anticipatory effects and 6 lagged effects:

$$\ln(Trade_{ijt}) = \sum_{\tau=-5}^6 \delta_{m\tau} RTAm_{\tau} + \sum_{\tau=-5}^6 \delta_{in\tau} RTAin_{\tau} + \sum_{\tau=-5}^6 \delta_{out\tau} RTAout_{\tau} + \beta_{ij} + \beta_{it} + \beta_{jt} + \epsilon_{ijt} \quad (4)$$

Our results, presented in table 5, suggest that the effects of RTA on trade creation within the region are long-lasting. The anticipatory effect is strong for RTAin. This reminds us that trade negotiations is a lengthy process. The decrease in trade inflows associated with a trade agreement occurs mainly in anticipation of the trade agreement. For the outflows, we notice some anticipation effects and a strong positive effect post implementation.

Figure 3. Results with GVCs



4 Quantitative exploration

4.1 Model setup

We follow [de Soyres and Gaillard \(2019\)](#) in modelling global value chains, with firms engaged in both importing and exporting intermediate inputs in a multi-country model of intra-industry trade with heterogeneous firms à la [Chaney \(2008\)](#) and [Melitz \(2003\)](#). There are J countries⁹, indexed by j or i .

Production technology

In any country, production is performed by a mass N of heterogeneous firms combining labor ℓ and a bundle M of intermediate inputs bought from other firms. Firms' productivity is the product of an idiosyncratic part φ and a country specific part Z . The production function for a firm with productivity φ is:

$$Y = Z \cdot \varphi \cdot \ell^\alpha \cdot M^{1-\alpha} .$$

Firms draw their productivity φ from a Pareto distribution with shape parameter γ and support $[\underline{\gamma}, \infty)$. The bundle of intermediate inputs M is a CES aggregate of intermediate inputs

⁹All parameters of the model are country specific except for the elasticity of substitution in the use of inputs in production and in the household's preferences.

Time	log(trade)		
	RTAm	RTAin	RTAout
-5	-0.0056 (-0.20)	-0.1614*** (-12.36)	0.0601*** (4.87)
-4	0.0014 (0.05)	-0.1615*** (-12.40)	0.0308* (2.46)
-3	-0.0080 (-0.26)	-0.1376*** (-10.29)	0.0291* (2.25)
-2	0.0429 (1.31)	-0.1073*** (-7.50)	0.0618*** (4.41)
-1	0.0737* (2.17)	-0.0735*** (-4.81)	0.0747*** (4.91)
0	0.1040** (2.99)	-0.0615*** (-3.83)	0.0938*** (5.74)
1	0.2019*** (5.49)	-0.0042 (-0.27)	0.1385*** (8.67)
2	0.2405*** (6.41)	-0.0150 (-1.01)	0.1423*** (9.46)
3	0.2429*** (6.12)	0.0023 (0.17)	0.1329*** (9.31)
4	0.3067*** (7.55)	-0.0093 (-0.67)	0.1213*** (8.78)
5	0.3287*** (7.96)	0.0287* (2.16)	0.1063*** (7.98)
6 and more	0.5777*** (0.1360)	0.0962*** (5.00)	0.2773*** (13.95)
Country pair FE	————— Yes —————		
Country × time FE	————— Yes —————		
Observations	————— 796,107 —————		
R ²	————— 0.799 —————		

t statistics in parentheses.

* p<0.05, ** p<0.01, *** p<0.001

Table 5. Timing of the effects of RTA on total trade flows

defined as follows:

$$M = \left(\int_{\omega \in \Omega} m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω is the set of indices of varieties to which the firm has access, and $m(\omega)$ denotes the use of variety ω . The elasticity of substitution used by firms to aggregate their inputs is parametrized by σ . We assume that $\sigma > 1$ to capture that firms benefit from accessing a broader set of varieties of intermediate inputs. The set Ω gathers the indices of all varieties produced domestically and supplied by foreign producers. It is the result of domestic and foreign firms' supply decisions and summarizes the endogenous global network of value

chains.

Each producer owns a blueprint to produce a single differentiated variety over which it has monopoly power. Each firm hires labor at a wage w and purchases bundles of intermediate inputs at price \mathcal{P} to minimize its cost of production. Define the price index for a bundle of intermediate inputs composed of varieties with indices in Ω as follows:

$$\mathcal{P}(\Omega) = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}.$$

where $p(\omega)$ denotes the price of variety ω . The CES structure implies a constant marginal cost per efficiency unit of the firm:

$$c(\Omega) = \frac{1}{Z} \cdot \frac{w^\alpha \cdot \mathcal{P}(\Omega)^{1-\alpha}}{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha}}. \quad (5)$$

The marginal cost of a firm with idiosyncratic productivity φ is $\frac{c(\Omega)}{\varphi}$. The notation highlights the dependence of the price index on the set of available varieties. This allows to trace the economic mechanisms through which the global network of value chains matters for economic outcomes.

The cost of supplying a country entails a fixed cost and a variable shipping cost. The fixed cost f_{ij} of serving country j from country i is paid in units of the production bundle. Shipping a unit of manufactured variety from country i to country j requires an “iceberg” variable cost τ_{ij} and $\tau_{ii} = 1$.

A firm exercises its monopoly power by maximizing profits given the CES demand schedule for intermediate inputs it faces. A firm with idiosyncratic productivity φ in country i supplying firms country j charges a constant markup over its marginal cost:¹⁰

$$p_{ij}(\varphi) = \tau_{ij} \cdot \frac{\sigma}{\sigma - 1} \cdot \frac{c(\Omega_i)}{\varphi}. \quad (6)$$

We identify the index of a variety with the idiosyncratic productivity with which the firm produces it and the country of production.

Following [Chaney \(2008\)](#), each country also produces a homogeneous good using a linear production technology requiring labor only. The homogeneous good sector absorbs a constant share $1 - \beta$ of the economy’s spending. The homogeneous good is freely traded. Its sector is competitive and large enough to pin down the wage.

¹⁰Note that the price charged to firms and consumers can only differ because of different elasticity of substitutions in firms’ and consumers’ CES aggregates, leading to different markups. This can be seen by comparing (6) and (7)

Household preferences and endowments

Each country is inhabited by a mass L of households. Each household inelastically supplies one unit of labor and has preferences over consumption of a homogeneous good and consumption bundles according to the following utility function:

$$U = q^{1-\beta} \times \left(\int_{\Omega} q(\omega)^{\frac{\sigma_f-1}{\sigma_f}} \right)^{\frac{\sigma_f}{\sigma_f-1} \times \beta},$$

where Ω denotes the endogenous set of all varieties to which the consumer has access and $q(\omega)$ denotes the consumption of variety ω . Consumers' propensity to substitute different varieties is given by σ_f and they have a preference for having access to a broader set of varieties $\sigma_f > 1$. The consumer maximizes its utility subject to the following budget constraint:

$$p \cdot q + \int_{\omega \in \Omega} p(\omega) \cdot q(\omega) d\omega = w \cdot N + \Pi + T_k.$$

where T_k is the total revenue levied on country k 's imports and Π denotes the aggregate profits to which the consumer is entitled from ownership of the firms in its country.

The CES demand structure implies that a firm with idiosyncratic productivity φ in country i supplying its final good to country j charges a constant markup over its marginal cost

$$p_{ij}^f(\varphi) = \tau_{ij} \cdot \frac{\sigma_f}{\sigma_f - 1} \cdot \frac{c(\Omega_i)}{\varphi}. \quad (7)$$

Global value chains

Firms demand varieties of intermediate inputs produced both at home and abroad. In turn firms choose which countries to serve based on their anticipated profits from sales of its variety of intermediate good and of final good. The global network of value chains results from firms' export strategies, as summarized by $(\Omega_j)_{j=1}^J$ which captures the availability of intermediate inputs around the world.

Each firm controls, through its demand for a given variety, the last stage of its production process. This assumption greatly simplifies the computation of equilibrium while retaining the essence of the aggregate implications of GVC, namely the inter-dependencies of firms' cost functions through the global network of value chains.

Unlike in [Antràs et al. \(2017\)](#), we bypass the complex discrete-choice decision of location

for each stage of production. This allows us to keep our model tractable and to focus on the aggregate consequences of international interdependence in production.

Firms' cost functions depend on the global value chain as made explicit in equation (5). The key determinants of the role of global value chains in firms' cost function is the share of expenditure on intermediate inputs in production and the extent to which intermediate inputs are substitutable. A larger share $1 - \alpha$ spent on intermediate inputs or a lower elasticity of substitution σ amplify the role played by global value chains in determining a firms' cost function.

Equilibrium

An equilibrium is a global network of value chains and a system of price indices for bundles of intermediate inputs and consumption, such that, firms export if and only if it is profitable for them to do so and price indices are CES aggregates of optimal pricing by monopolists.

The competitive homogeneous good sector clears the labor market as a residual and ensures that global trade is balanced.

In characterizing the optimal pricing and input use of firms, we have already partially characterized the equilibrium of the model. Profits are strictly increasing in a firm's idiosyncratic productivity. As a result, the decision for firms in i to export to j can be characterized by a threshold productivity above which firms find it profitable to export.

The GVCs creates interdependencies across firms' cost functions. Since monopolists charge a constant markup over their marginal cost, these interdependencies can be summarized in a system of J equations in J ideal price indices of a bundle of available intermediate inputs. An equilibrium is then fully characterized by a fixed point of a mapping of bilateral threshold productivities at which a firm breaks-even by exporting. All equilibrium conditions as well as our algorithm to compute the allocation are described in details in Appendix A.

4.2 Comparative statics of regional trade agreements

We study the effect of a regional trade agreement in a world composed of three countries, or trade blocks, A, B and C. In particular, we study the equilibrium response following a regional trade agreement between countries A and B and the change such agreement produces on prices, costs, trade flows and firms' selection into supplying foreign markets among countries A, B, and C.

We compare the results from the numerical exercise to our empirical findings that RTAs are associated with: i) an increase in trade within the region; ii) a decrease in inflows to the

region; iii) an increase in outflows from the region.

For the purpose of this numerical exercise, we use the following parametrization: trade frictions are an iceberg cost of 20%; that is $\tau_{ij} = 1.2$. Trade liberalization between countries A and B is modelled as a drop in trade frictions from 20% to 10%. We set the elasticity of substitution for final goods and intermediate inputs to the same value $\sigma_c = \sigma_f = 5$. The share of utility from differentiated goods is $\beta = 0.75$. Countries in the region are symmetric and the size of the region composed of A and B is 25% of the world. For each country, the mass of potential firms is 10% of the size of the country. Firms draw their productivity from a Pareto distribution with scale 1 and tail index $\gamma = 4.6 (= \sigma - 0.4)$. The overall efficiency is normalized to $Z = 1$ which pins down the wage to $w = 1$ across all three countries. Fixed costs of serving the domestic market is set at $f_{ii} = 0.7$ while the fixed cost of serving the foreign market is set at $f_{ij} = 0.9$ for $i \neq j$.

Note that in the initial situation, all three countries are identical. The equilibrium is symmetric. Below, we report the equilibrium post regional trade agreement and the percentage change induced by the regional trade agreement.

Regional trade agreements in a model with GVC

We start our investigations by looking at the prediction of our model with input-output linkages. For this numerical exercise, we set the labor share α to 0.5. The numerical results are the following. Upon trade liberalization, sourcing from the partner within the region increases; total inflows to the region decrease; total outflows from the region increase. This is the net balance of a decrease in exports of intermediates and an increase in exports of final goods to country C.

Total trade flows of the initial equilibrium are reported in the matrix T^t and the percentage change in trade flows is reported in the matrix ΔT^t .

$$\begin{bmatrix} T_{AA}^t & T_{AB}^t & T_{AC}^t \\ T_{BA}^t & T_{BB}^t & T_{BC}^t \\ T_{CA}^t & T_{CB}^t & T_{CC}^t \end{bmatrix} = \begin{bmatrix} 4.3 & 2.0 & 2.0 \\ 2.0 & 4.3 & 2.0 \\ 2.0 & 2.0 & 4.3 \end{bmatrix} \text{ and } \Delta T^t = \begin{bmatrix} -6.8\% & 28\% & 2.1\% \\ 28\% & -6.8\% & 2.1\% \\ -12\% & -12\% & -3.3\% \end{bmatrix}.$$

Total trade flows (denoted T^t) are the sum of trade in goods consumed by consumers (denoted T^c) and in intermediate inputs used by firms (denoted T^f). We now decompose the above matrices of trade flows and percentage changes into trade for consumers and firms.

$$\begin{bmatrix} T_{AA}^f & T_{AB}^f & T_{AC}^f \\ T_{BA}^f & T_{BB}^f & T_{BC}^f \\ T_{CA}^f & T_{CB}^f & T_{CC}^f \end{bmatrix} = \begin{bmatrix} 0.28 & 0.13 & 0.13 \\ 0.13 & 0.28 & 0.13 \\ 0.13 & 0.13 & 0.28 \end{bmatrix} \text{ and } \Delta T^f = \begin{bmatrix} -3.7\% & 32\% & -4.8\% \\ 32\% & -3.7\% & -4.8\% \\ -8.8\% & -8.8\% & -9.8\% \end{bmatrix}.$$

$$\begin{bmatrix} T_{AA}^c & T_{AB}^c & T_{AC}^c \\ T_{BA}^c & T_{BB}^c & T_{BC}^c \\ T_{CA}^c & T_{CB}^c & T_{CC}^c \end{bmatrix} = \begin{bmatrix} 4.0 & 1.9 & 1.9 \\ 1.9 & 4.0 & 1.9 \\ 1.9 & 1.9 & 4.0 \end{bmatrix} \text{ and } \Delta T^c = \begin{bmatrix} -7\% & 27\% & 2.6\% \\ 27\% & -7\% & 2.6\% \\ -12\% & -12\% & -2.8\% \end{bmatrix}.$$

We note that trade patterns of trade are mainly coming from trade in final goods, which is an order of magnitude larger than trade in intermediates. Despite trade in intermediates having little impact on patterns of total trade flows, the gains from trade at the firm level have a sizable impact on the pattern of total trade flows. This last point will become clearer when we evaluate the importance of input-output linkages in the next subsection.

- *Costs*

The marginal cost per unit of efficiency drops more for countries within the region. For members, the marginal cost decreases by 1.05% whereas for the rest of the world, the marginal cost decreases by 0.07%. Note that without input-output linkages ($\alpha = 1$), the marginal cost would be constant.

- *Firm selection into servicing domestic and foreign markets*

The initial equilibrium is symmetric. In each country there is a threshold above which firms service the domestic market and a higher threshold above which firms also exports. Upon trade liberalization, within the region, fewer firms service the domestic market but more firms service the foreign market. This is the standard effect emphasized in Melitz (2003). There are also fewer firms in C finding it profitable to export to the region. Last, there are more firms within the region finding it profitable to export to C.

$$\begin{bmatrix} \varphi_{AA} & \varphi_{AB} & \varphi_{AC} \\ \varphi_{BA} & \varphi_{BB} & \varphi_{BC} \\ \varphi_{CA} & \varphi_{CB} & \varphi_{CC} \end{bmatrix} = \begin{bmatrix} 1.3 & 1.8 & 1.8 \\ 1.8 & 1.3 & 1.8 \\ 1.8 & 1.8 & 1.3 \end{bmatrix} \text{ and } \Delta \varphi = \begin{bmatrix} 1.3\% & -7.1\% & -1.2\% \\ -7.1\% & 1.3\% & -1.2\% \\ 3.3\% & 3.3\% & 0.7\% \end{bmatrix}.$$

In terms of aggregate trade flows, we conclude that the model with input-output linkages is successful at qualitatively replicating our empirical findings.

Regional trade agreements in a model *without* GVC

We continue our investigations by computing the comparative statics in a benchmark model *without* use for intermediate inputs in production.

Our model nests a model with trade in final good only, as in the Chaney-Melitz model. It suffices to set the share of intermediate inputs in production to 1.

To that end, we compute the comparative statics exercise with the same parameter values with the exception that the labor share is $\alpha = 1$ so that intermediate inputs are not used in production. In this limiting case, the production cost does not depend on the pattern of trade and equation (5) becomes $c = \frac{1}{2}w$.

The main lesson from this numerical exercise is that the equilibrium response is qualitatively in line with our empirical findings i) and ii) but not with iii). There is a negligible negative response of trade flows from the region to the rest of the world.

$$\begin{bmatrix} T_{AA}^t & T_{AB}^t & T_{AC}^t \\ T_{BA}^t & T_{BB}^t & T_{BC}^t \\ T_{CA}^t & T_{CB}^t & T_{CC}^t \end{bmatrix} = \begin{bmatrix} 4.0 & 1.9 & 1.9 \\ 1.9 & 4.0 & 1.9 \\ 1.9 & 1.9 & 4.0 \end{bmatrix} \text{ and } \Delta T^t = \begin{bmatrix} -8.1\% & 26\% & -0.1\% \\ 26\% & -8.1\% & -0.1\% \\ -8.1\% & -8.1\% & -0.1\% \end{bmatrix}.$$

Importance of initial regional integration

The numerical exercise consists in computing the same comparative statics as presented above, but performed with different initial conditions (before the regional integration). More precisely, we contrast our baseline results with the consequence of trade liberalization when it occurs within a region that is already more integrated than world integration. We consider two sources of greater integration: relatively low fixed cost and relatively low variable costs.

Initial region due lower fixed costs

We assume that $f(A, B) = f(B, A) = 0.35$ and otherwise, as before $f(i, i) = 0.3$ for $i = A, B, C$ and $f(i, C) = f(C, i) = 0.5$ for $i \neq C$. From this initial condition, reducing trade barriers between countries A and B leads to the following results:

$$\begin{bmatrix} T_{AA}^t & T_{AB}^t & T_{AC}^t \\ T_{BA}^t & T_{BB}^t & T_{BC}^t \\ T_{CA}^t & T_{CB}^t & T_{CC}^t \end{bmatrix} = \begin{bmatrix} 4.2 & 2.1 & 2.0 \\ 2.1 & 4.2 & 2.0 \\ 1.9 & 1.9 & 4.2 \end{bmatrix} \text{ and } \Delta T^t = \begin{bmatrix} -7.2\% & 27\% & 2.2\% \\ 27\% & -7.2\% & 2.2\% \\ -12\% & -12\% & -3.5\% \end{bmatrix}.$$

We see that higher initial integration through global value chains slightly enhances the percentage increase in outflows from the region. This is qualitatively in line with our empirical

findings, where we suggested that, upon signing a regional agreement, countries characterized by high regional integration *before* signing the RTA are associated with larger export gains to the rest of the world (i.e. larger *outflows*).

Initial region due lower variable costs

The numerical exercise reported in this section is based on the same parametrization with the exception that variable trade barriers between the region and the rest of the world are initially higher than within the region; that is:

$$\tau(1,3) = \tau(3,1) = \tau(2,3) = \tau(3,2) = 1.3. \quad (8)$$

$$\begin{bmatrix} T_{AA}^t & T_{AB}^t & T_{AC}^t \\ T_{BA}^t & T_{BB}^t & T_{BC}^t \\ T_{CA}^t & T_{CB}^t & T_{CC}^t \end{bmatrix} = \begin{bmatrix} 4.6 & 2.1 & 1.7 \\ 2.1 & 4.6 & 1.7 \\ 1.5 & 1.5 & 4.7 \end{bmatrix} \text{ and } \Delta T^t = \begin{bmatrix} -7.5\% & 27\% & 3\% \\ 27\% & -7.5\% & 3\% \\ -13\% & -13\% & -3.3\% \end{bmatrix}.$$

Again, a preexisting region due to higher variable trade barriers with the rest of the world heightens the percentage increase in outflows from the region.

5 Regionalism and Multilateralism

We now discuss the implications of our empirical and quantitative findings for regionalism and multilateralism. A situation in which a Regional Trade Agreement heightens (resp. dampens) the incentive for trade protection between the region and the rest of the world is a case of regionalism (resp. multilateralism).

In particular, we study the effect of the change in trade flows associated with a Regional Trade Agreement on the incentives to erect trade barriers between the region and the rest of the world. We draw from the literature on optimal trade policy which finds that the volume of trade increases the incentive to raise trade barriers (cf. [Bagwell and Staiger \(2011\)](#)).¹¹ Our separate identification of the change in both inflows and outflows associated with a RTA allows to discuss i) the incentives for the region to raise trade barriers on inflows from the rest of the world, and ii) the incentives for the rest of the world to raise trade barriers on outflows from the region.

First, we consider the incentives for the region to raise trade barriers on inflows from the rest of the world. Members of the RTA that are also WTO members are constrained in setting their external policy to levels that are less constraining than they were before the regional

¹¹We emphasize, like [Bagwell and Staiger \(1990\)](#) Section 5, that the relevant determinant of trade protection is the trade volume and not the trade balance.

agreement. Our finding that the region imports less from the rest of the world suggests that a regional trade agreement lowers the region's motive for trade protection from the rest of the world. This conclusion is in line with the empirical findings of [Estevadeordal et al. \(2008\)](#).

Less studied is the incentive for the rest of the world to raise trade barriers on outflows from the region in response to a Regional Trade Agreement. We find an increase in export volume from the region to the rest of the world for regions that have well developed regional value chains. Theories of optimal trade policy suggest then that the protectionist motive of the rest of the world toward the region is heightened.

It is important to note that our analysis is only suggestive of incentives for countries to alter their trade policy in response to a regional trade agreement. Empirical analysis of changes in trade protections in response to a Regional Trade Agreement, as in [Estevadeordal et al. \(2008\)](#), would be a natural complementary analysis to ours. While this is beyond the scope of this paper, our findings point to Global Value Chains as a determining factor for the incentives of non-members of a Regional Trade Agreement to change their trade policy in response to a RTA.

6 Conclusion

This paper analyses the consequences of Regional Trade Agreements on the changes in trade flows both within the agreement region and with the rest of the world. Based on the universe of all trade agreements between 1962 and 2014 for a total of 215 countries, our results show that, once we control for country-pair and country-time fixed effects, RTAs are associated with an increase in trade flows within the zone, and an increase of the outflows from the region towards the rest of the world. However, there is a decrease in inflows from the rest of the world towards the agreement region.

We then study the role of Global Value Chains in shaping the reorganization of trade flows after a trade agreement. Our analysis reveals that the increase in trade flows towards the rest of the world is associated with a stronger participation in regional value chains. When countries are not integrated into production chains, they experience a decrease in both imports from and exports to the rest of the world after signing a regional trade agreement. Overall, our findings show that Global Value Chains are associated with an important change in the way trade flows are affected by trade agreement: by allowing firms to decrease their input price and hence reduce their production cost, a regional trade agreement in a region that is well integrated in terms production is associated with marked gains in market shares in the rest of the world.

Finally, we propose a tractable model of international trade with multiple countries and

firms engaged in both import and export activities. Using a series of simple comparative statics, we show that global value chains are key in matching the data. In particular, models of trade in final goods cannot reproduce the increase in outflow from a region that signs a trade agreement. Our analysis allows us to discuss the consequences of global value chain integration for countries' incentive to form regional trade blocs as well as the trade-off between regionalism and multilateralism.

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A Theoretical Appendix

We present in detail here the steps needed to fully characterize an equilibrium of our model as well as the computation strategy used in our quantitative exercise. To clarify the role of different elasticities of substitution, we denote by σ_f and σ_c the elasticities used in firms' and consumers' aggregation bundle respectively.

Demand.

Since the elasticity of substitutions for final goods and intermediate inputs aggregation are not equal, we define two price index over varieties Ω_k , the set of all varieties available in country k . Price indices used by consumers and firms respectively, are defined as:

$$\mathcal{P}_{k,c} = \left(\int_{\Omega_k} p_k(v)^{1-\sigma_c} dv \right)^{\frac{1}{1-\sigma_c}}, \quad \mathcal{P}_{k,f} = \left(\int_{\Omega_k} p_k(v)^{1-\sigma_f} dv \right)^{\frac{1}{1-\sigma_f}}.$$

Given prices, total demand faced by the producer of a variety v when selling goods in country k is given by the sum of demand stemming from both the *final good* and the *intermediate input* markets:

$$q_k(v) = \left(\frac{p_{k,c}(v)}{\mathcal{P}_{k,c}} \right)^{-\sigma_c} \frac{\beta X_k}{\mathcal{P}_{k,c}} + \left(\frac{p_{k,f}(v)}{\mathcal{P}_{k,f}} \right)^{-\sigma_f} M_k \quad (9)$$

where we defined $X_k = w_k L_k + \Pi_k$ as total consumers' spending in country k . Let S_k denote total firms' spending (on labor and intermediates to cover both fixed and variable costs of production) in country k . The CES demand implies $M_k = \frac{(1-\alpha)S_k}{\mathcal{P}_{k,f}}$.

Pricing.

Prices are a constant markup over marginal cost $c(\varphi)$. In any country k , we have:

$$c(\varphi) = c_k \frac{1}{\varphi} \quad \text{and} \quad c_k = \frac{1}{Z_k} \frac{w_k^\alpha \mathcal{P}_{k,f}^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}. \quad (10)$$

The price of a firm from k with productivity φ selling its good in country i is:

$$p_{ki,c}(\varphi) = \tau_{ki} \frac{\sigma_c}{\sigma_c - 1} \frac{c_k}{\varphi} \quad p_{ki,f}(\varphi) = \tau_{ki} \frac{\sigma_f}{\sigma_f - 1} \frac{c_k}{\varphi}$$

Denote by $\overline{\varphi}_{ki}$ the productivity cutoff above which a firm from country k exports to i . We give an exact expression of these thresholds below.

Price index for firms. Due to input-output linkages, the prices of intermediate inputs

solves a system of equations. Price indices in all countries must jointly solve:

$$\begin{aligned}
\mathcal{P}_{k,f}^{1-\sigma_f} &= \sum_j M_j \int_{\underline{\varphi}_{jk}}^{\infty} p_{jk,f}(\varphi)^{1-\sigma_f} g_j(\varphi) d\varphi \\
&= \sum_j M_j \int_{\underline{\varphi}_{jk}}^{\infty} \left(\tau_{jk} \frac{\sigma_f}{\sigma_f - 1} \frac{1}{Z_j} \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} w_j^\alpha \mathcal{P}_{j,f}^{1-\alpha} \frac{1}{\varphi} \right)^{1-\sigma_f} g_j(\varphi) d\varphi \\
\text{defining } \mu_{j,f} &\equiv \frac{\sigma_f}{\sigma_f - 1} \frac{w_j^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \\
&= \sum_j M_j \left(\frac{\mu_{j,f}}{Z_j} \right)^{1-\sigma_f} \left(\mathcal{P}_{j,f}^{1-\sigma_f} \right)^{1-\alpha} \tau_{ji}^{1-\sigma_f} \int_{\underline{\varphi}_{jk}}^{\infty} \left(\frac{1}{\varphi} \right)^{1-\sigma_f} g_j(\varphi) d\varphi \\
&= \sum_j \frac{\gamma_j}{\gamma_j + 1 - \sigma_f} M_j \left(\frac{\mu_{j,f}}{Z_j} \right)^{1-\sigma_f} \tau_{ji}^{1-\sigma_f} \underline{\varphi}_j^{\gamma_j} \overline{\varphi}_{jk}^{-(\gamma_j+1-\sigma_f)} \left(\mathcal{P}_{j,f}^{1-\sigma_f} \right)^{1-\alpha}
\end{aligned}$$

where $\underline{\varphi}_j$ is the lower bound of the Pareto distribution of idiosyncratic productivity in country j . It is convenient to define the price index of all varieties of intermediate goods exported from k to j as follows:

$$\mathcal{P}_{kj,f}^{1-\sigma_f} = M_k \int_{\underline{\varphi}_{kj}}^{\infty} p_{kj,f}(\varphi)^{1-\sigma_f} g(\varphi) d\varphi .$$

In the derivations above, we used the fact that there is a simple relationship between the price index defined over varieties from k sold to other domestic firms ($\mathcal{P}_{k,f}$) and varieties from k exported to country i :

$$\mathcal{P}_{ki,f}^{1-\sigma_f} = \frac{\gamma_k}{\gamma_k + 1 - \sigma_f} \mathcal{M}_k \left(\frac{\mu_{k,f}}{Z_k} \right)^{1-\sigma_f} \tau_{ki}^{1-\sigma_f} \underline{\varphi}_k^{\gamma_k} \varphi_{ki}^{-(\gamma_k+1-\sigma_f)} \left(\mathcal{P}_{k,f}^{1-\sigma_f} \right)^{1-\alpha} .$$

From now on, we normalize the location parameter of the productivity distribution and assume that $\underline{\varphi}_j = 1$ for all countries. Note that we will need to make sure that all thresholds are above 1 in our computations. After a change of variable $x_{k,f} = \mathcal{P}_{k,f}^{1-\sigma}$, the system reads:

$$x_{k,f} = \sum_j \lambda_j \tau_{jk}^{1-\sigma_f} \left(\overline{\varphi}_{jk}^{-(\gamma_j+1-\sigma_f)} \right) x_{j,f}^{1-\alpha}, \quad x_{k,f} \neq 0 \text{ for all } k, \quad (11)$$

where $\lambda_j = \frac{\gamma_j}{\gamma_j+1-\sigma_f} M_j \left(\frac{\mu_j}{Z_j} \right)^{1-\sigma_f}$. We write this system of equations in matrix notation as:

$$\mathbf{x} = \mathbf{P} \mathbf{x}^{1-\alpha}, \quad \mathbf{x} \neq 0$$

This system of equations has a unique non negative solution.

$$\mathbf{x} = \begin{bmatrix} x_{1,f} \\ x_{2,f} \\ \vdots \\ x_{N,f} \end{bmatrix} \quad \text{and} \quad \mathbf{P} = \begin{bmatrix} \lambda_1 \tau_{11}^{1-\sigma_f} \underline{\varphi}_{11}^{-(\gamma_1+1-\sigma_f)} & \lambda_2 \tau_{21}^{1-\sigma_f} \underline{\varphi}_{21}^{-(\gamma_2+1-\sigma_f)} & \dots & \lambda_N \tau_{N1}^{1-\sigma_f} \underline{\varphi}_{N1}^{-(\gamma_N+1-\sigma_f)} \\ \lambda_1 \tau_{12}^{1-\sigma_f} \underline{\varphi}_{12}^{-(\gamma_1+1-\sigma_f)} & \lambda_2 \tau_{22}^{1-\sigma_f} \underline{\varphi}_{22}^{-(\gamma_2+1-\sigma_f)} & \dots & \lambda_N \tau_{N2}^{1-\sigma_f} \underline{\varphi}_{N2}^{-(\gamma_N+1-\sigma_f)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_1 \tau_{1N}^{1-\sigma_f} \underline{\varphi}_{1N}^{-(\gamma_1+1-\sigma_f)} & \lambda_2 \tau_{2N}^{1-\sigma_f} \underline{\varphi}_{2N}^{-(\gamma_2+1-\sigma_f)} & \dots & \lambda_N \tau_{NN}^{1-\sigma_f} \underline{\varphi}_{NN}^{-(\gamma_N+1-\sigma_f)} \end{bmatrix}$$

Price index for consumers.

$$\begin{aligned} \mathcal{P}_{k,c}^{1-\sigma_c} &= \sum_j M_j \int_{\underline{\varphi}_{jk}}^{\infty} p_{jk,c}(\varphi)^{1-\sigma_c} g_j(\varphi) d\varphi \\ &= \sum_j M_j \int_{\underline{\varphi}_{jk}}^{\infty} \left(\tau_{jk} \frac{\sigma_c}{\sigma_c - 1} \frac{1}{Z_j} \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} w_j^\alpha \mathcal{P}_{j,f}^{1-\alpha} \frac{1}{\varphi} \right)^{1-\sigma_c} g_j(\varphi) d\varphi \\ \text{defining } \mu_{j,c} &\equiv \frac{\sigma_c}{\sigma_c - 1} \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} w_j^\alpha \\ &= \sum_j M_j \left(\frac{\mu_{j,c}}{Z_j} \right)^{1-\sigma_c} \left(\mathcal{P}_{j,f}^{1-\sigma_c} \right)^{1-\alpha} \tau_{jk}^{1-\sigma_c} \int_{\underline{\varphi}_{jk}}^{\infty} \left(\frac{1}{\varphi} \right)^{1-\sigma_c} g_j(\varphi) d\varphi \\ &= \sum_j \frac{\gamma_j}{\gamma_j + 1 - \sigma_c} M_j \left(\frac{\mu_{j,c}}{Z_j} \right)^{1-\sigma_c} \tau_{jk}^{1-\sigma_c} \underline{\varphi}_j^{\gamma_j} \underline{\varphi}_{jk}^{-(\gamma_j+1-\sigma_c)} \left(\mathcal{P}_{j,f}^{1-\sigma_c} \right)^{1-\alpha} \\ &= \sum_j \lambda_{j,c} \tau_{jk}^{1-\sigma_c} \underline{\varphi}_j^{\gamma_j} \underline{\varphi}_{jk}^{-(\gamma_j+1-\sigma_c)} \left(\mathcal{P}_{j,f}^{1-\sigma_c} \right)^{1-\alpha} \end{aligned}$$

where the derivations follow the ones for the price index for firms and $\mu_{j,c} \equiv \frac{\sigma_c}{\sigma_c - 1} \frac{w_j^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$

$$\lambda_{j,c} = \frac{\gamma_j}{\gamma_j + 1 - \sigma_c} M_j \left(\frac{\mu_{j,c}}{Z_j} \right)^{1-\sigma_c}.$$

Profits and revenues.

Using standard results, we can also write *total* revenues a firm from k with productivity φ is earning when selling in country i :

$$\begin{aligned} r_{ki,c}(\varphi) &= \left(\frac{\sigma_c}{\sigma_c - 1} \tau_{ki} c_k \right)^{1-\sigma_c} \left(\frac{1}{\varphi} \right)^{1-\sigma_c} \mathcal{P}_{i,c}^{\sigma_c-1} \cdot \beta X_i \\ r_{ki,f}(\varphi) &= \left(\frac{\sigma_f}{\sigma_f - 1} \tau_{ki} c_k \right)^{1-\sigma_f} \left(\frac{1}{\varphi} \right)^{1-\sigma_f} \mathcal{P}_{i,f}^{\sigma_f-1} \cdot (1-\alpha) S_i \\ r_{ki}(\varphi) &= r_{ki,c}(\varphi) + r_{ki,f}(\varphi) \end{aligned}$$

where S_i is equal to total firms' spending in i and includes spending on both variable and

fixed costs. With a constant markup of $\sigma/(\sigma - 1)$, gross profits are a fraction $(1/\sigma)$ of sales. The profits of a firm in k with productivity φ from selling in country i are:

$$\begin{aligned}\pi_{ki,c}(\varphi) &= \frac{1}{\sigma_c} r_{ki,c}(\varphi) , \\ \pi_{ki,f}(\varphi) &= \frac{1}{\sigma_f} r_{ki,f}(\varphi) , \\ \pi_{ki}(\varphi) &= \pi_{ki,c}(\varphi) + \pi_{ki,f}(\varphi).\end{aligned}$$

Threshold productivity. The fixed costs of servicing market i from country k is f_{ki} units of the input bundle which costs $c_k = \frac{1}{Z_k} \frac{w_k^\alpha \mathcal{P}_{k,f}^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$.

$$\begin{aligned}f_{ki} c_k &= \pi_{ki,c}(\varphi_{ki}) + \pi_{ki,f}(\varphi_{ki}) \\ &= \frac{1}{\sigma_c} \left(\frac{\sigma_c}{\sigma_c - 1} \tau_{ki} c_k \right)^{1-\sigma_c} \left(\frac{1}{\varphi_{ki}} \right)^{1-\sigma_c} \mathcal{P}_{i,c}^{\sigma_c-1} \cdot \beta X_i + \frac{1}{\sigma_f} \left(\frac{\sigma_f}{\sigma_f - 1} \tau_{ki} c_k \right)^{1-\sigma_f} \left(\frac{1}{\varphi_{ki}} \right)^{1-\sigma_f} \mathcal{P}_{i,f}^{\sigma_f-1} \cdot (1-\alpha) S_i\end{aligned}$$

Aggregate profits, net of fixed costs are:

$$\Pi_k = \sum_i \frac{1}{\sigma_c} R_{ki,c} + \frac{1}{\sigma_f} R_{ki,f} - \mathcal{M}_k c_k f_{ki} \underline{\varphi}_k^{\gamma_k} \overline{\varphi}_{ki}^{-\gamma_k}$$

The following result will be convenient for the computation of equilibria under the assumption that elasticities are the same:

Lemma 1: Suppose that elasticities of substitution are equal : $\sigma_c = \sigma_f = \sigma$. Then total profits net of fixed cost payment in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma_k \sigma} R_k$$

Proof: For simplicity, we write the proof in the case with $\gamma_k = \gamma$. First, total profits net of fixed costs for all firms in k can be written $\Pi_k = \frac{R_k}{\sigma} - \sum_j FC_{k \rightarrow j}$, where $FC_{k \rightarrow j}$ is the sum of fixed cost payment from all firms from country k serving market j . Then, note that total fixed cost payment for all firms in country i is:

$$FC_{k \rightarrow j} = M_k \int_{\frac{c_k}{\overline{\varphi}_{kj}}}^{+\infty} f_{kj} \times \frac{c_k}{Z_k} \times \gamma \varphi^{-\gamma-1} \times d\varphi = M_k f_{kj} \frac{c_k}{Z_k} \times \overline{\varphi}_{kj}^{-\gamma}$$

For all k, j , total revenues (sales) from k to j can be written as:

$$\begin{aligned} R_{kj} &= M_k \int_{\overline{\varphi}_{kj}}^{+\infty} \left(\tau_{kj} \frac{\sigma}{\sigma-1} \frac{c_k}{Z_k} \frac{1}{P_j} \right)^{1-\sigma} \times \left[(1-\alpha)S_j + \beta X_j \right] \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{\gamma M_k}{\gamma - (\sigma - 1)} \times \left(\tau_{kj} \frac{\sigma}{\sigma-1} \frac{c_k}{Z_k} \frac{1}{P_j} \right)^{1-\sigma} \times \left[(1-\alpha)S_j + \beta X_j \right] \overline{\varphi}_{kj}^{\sigma-\gamma-1} \end{aligned}$$

Next, using the expression for $\overline{\varphi}_{kj}$ in (20), we get:

$$R_{kj} = \frac{\gamma M_k}{\gamma - (\sigma - 1)} \times \sigma f_{kj} \frac{c_k}{Z_k} \overline{\varphi}_{kj}^{-\gamma} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{k \rightarrow j}$$

Combining those expressions, we get

$$\sum_j FC_{k \rightarrow j} = \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times \left(\sum_j R_{kj} \right) = \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times R_k$$

Using this expression of $\sum_j FC_{k \rightarrow j}$ in the definition of profits completes the proof.

Revenues.

We can express total revenues earned by all firms in k as:

$$\begin{aligned} R_k &= \sum_j R_{kj,c} + R_{kj,f} , \\ R_{kj,c} &= \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta X_j \\ R_{kj,f} &= \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha)S_j . \end{aligned} \tag{12}$$

Thresholds.

Recall that $\overline{\varphi}_{ki}$ denotes the productivity cutoff above which a firm from country k finds it profitable to export to i . It is characterized by the following condition:

$$\pi_{ki}(\overline{\varphi}_{ki}) = \frac{c_k}{Z_k} f_{ki} . \tag{13}$$

Since profits are a fraction $\frac{1}{\sigma}$ of sales, we get:

$$\frac{1}{\sigma_c} r_{ki,c}(\overline{\varphi}_{ki}) + \frac{1}{\sigma_f} r_{ki,f}(\overline{\varphi}_{ki}) = \frac{c_k}{Z_k} f_{ki} .$$

Equilibrium

Given equilibrium threshold productivities, aggregate equilibrium variables are a solution to the following system of equations:

$$\left\{ \begin{array}{l} \mathbf{x} = \mathbf{P} \mathbf{x}^{1-\alpha}, \quad \mathbf{x} \neq 0, \quad x_{k,f} \equiv \mathcal{P}_{k,f}^{1-\sigma} \\ FC_k = \mathcal{M}_k c_k f_{ki} \underline{\varphi}_k^{\gamma_k} \varphi_{ki}^{-\gamma_k} \\ X_k = w_k L_k + \Pi_k + T_k \quad \text{for all } k \\ S_k = R_k - \Pi_k \quad \text{for all } k \\ R_k = \sum_j R_{kj,c} + R_{kj,f} \quad \text{for all } k \\ \Pi_k = \sum_i \frac{1}{\sigma_c} R_{ki,c} + \frac{1}{\sigma_f} R_{ki,f} - FC_k \\ R_{kj,f} = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) S_j \\ R_{kj,c} = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta X_j \end{array} \right.$$

There are two ways to go about solving for a tentative equilibrium given threshold productivities:

A Solve for the bilateral revenues $R_{kj,f}$ and $R_{kj,c}$. This is a system of $2 * N^2$ equations. If elasticities are the same, then it reduces to a system of N equations.

$$\left\{ \begin{array}{l} \mathbf{x} = \mathbf{P} \mathbf{x}^{1-\alpha}, \quad \mathbf{x} \neq 0, \quad x_{k,f} \equiv \mathcal{P}_{k,f}^{1-\sigma} \\ FC_k = \mathcal{M}_k c_k f_{ki} \underline{\varphi}_k^{\gamma_k} \varphi_{ki}^{-\gamma_k} \\ R_{kj,f} = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) \left(\sum_i \frac{\sigma_c-1}{\sigma_c} R_{ji,c} + \frac{\sigma_f-1}{\sigma_f} R_{ji,f} + FC_k \right) \\ R_{kj,c} = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta \left(w_j L_j + \sum_i \frac{1}{\sigma_c} R_{ji,c} + \frac{1}{\sigma_f} R_{ji,f} - FC_k \right) \\ R_j = \sum_i R_{ji,c} + R_{ji,f} \quad \text{for all } k \\ \Pi_j = \sum_i \frac{1}{\sigma_c} R_{ji,c} + \frac{1}{\sigma_f} R_{ji,f} - FC_k \\ X_k = w_k L_k + \Pi_k + T_k \quad \text{for all } k \\ S_k = R_k - \Pi_k \quad \text{for all } k \end{array} \right.$$

B Solve for the country level profits and revenues Π_k and R_k . This is a system of $2 * N$

equations. If elasticities are the same, then it reduces to a system of N equations.

$$\left\{ \begin{array}{l} \mathbf{x} = \mathbf{P} \mathbf{x}^{1-\alpha}, \quad \mathbf{x} \neq \mathbf{0}, \quad x_{k,f} \equiv \mathcal{P}_{k,f}^{1-\sigma} \\ FC_k = \mathcal{M}_k c_k f_{ki} \varphi_k^{\gamma_k} \varphi_{ki}^{-\gamma_k} \\ X_k = w_k L_k + \Pi_k \quad \text{for all } k \\ S_k = R_k - \Pi_k \quad \text{for all } k \\ R_k = \sum_j \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_c} \beta X_j + \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) S_j \quad \text{for all } k \\ \Pi_k = \sum_i \frac{1}{\sigma_c} \left(\frac{P_{ki,f}}{P_{i,f}} \right)^{1-\sigma_c} \beta X_i + \frac{1}{\sigma_f} \left(\frac{P_{ki,f}}{P_{i,f}} \right)^{1-\sigma_f} (1-\alpha) S_i - FC_k \\ R_{kj,f} = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) S_j \\ R_{kj,c} = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_c} \beta X_j \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathbf{x} = \mathbf{P} \mathbf{x}^{1-\alpha}, \quad \mathbf{x} \neq \mathbf{0}, \quad x_{k,f} \equiv \mathcal{P}_{k,f}^{1-\sigma} \\ FC_k = \mathcal{M}_k c_k f_{ki} \varphi_k^{\gamma_k} \varphi_{ki}^{-\gamma_k} \\ X_k = w_k L_k + \Pi_k \quad \text{for all } k \\ S_k = R_k - \Pi_k \quad \text{for all } k \\ R_k = \sum_j \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta (w_j L_j + \Pi_j) + \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) (R_j - \Pi_j) \quad \text{for all } k \\ \Pi_k = \sum_i \frac{1}{\sigma_c} \left(\frac{P_{ki,c}}{P_{i,c}} \right)^{1-\sigma_c} \beta (w_i L_i + \Pi_i) + \frac{1}{\sigma_f} \left(\frac{P_{ki,f}}{P_{i,f}} \right)^{1-\sigma_f} (1-\alpha) (R_i - \Pi_i) - FC_k \end{array} \right.$$

We can solve this system of equations sequentially by first solving for prices and then solving for revenues and profits:

$$\begin{aligned} \Pi_k &= \sum_j \left(\frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) \right) \Pi_j + \sum_j \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) R_j \\ &\quad + \sum_j \frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j - FC_k \\ R_k &= \sum_j \left(\left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) \right) \Pi_j + \sum_j \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1-\alpha) R_j \\ &\quad + \sum_j \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j \end{aligned}$$

In matrix form, this system can be written as:

$$\begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} \mathbf{1} + \begin{bmatrix} \mathbf{B}^1 & \mathbf{B}^2 \\ \mathbf{B}^3 & \mathbf{B}^4 \end{bmatrix} \begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix}$$

Where elements of matrices **A** and **B** are defined as:

$$A_{kj}^1 = \frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j - FC_k \quad \text{and} \quad A_{kj}^2 = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j$$

and

$$B_{kj}^1 = \frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{kj}^2 = \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{ki}^3 = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{ki}^4 = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

Finally, gathering terms for revenues and profit, we get a simple system:

$$[\mathbf{I} - \mathbf{B}] \begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix} = \mathbf{A} \mathbf{1} \quad (14)$$

where matrices $\mathbf{\Pi}$ and \mathbf{R} gather all revenues and profits.

$$\mathbf{\Pi} = \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_N \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Special Case with Equal elasticities $\sigma_f = \sigma_c$

If elasticities are equal, using lemma 1 above and the fact that $R_k = S_k + \Pi_k$, we get:

$$S_k = \left(\frac{\sigma \gamma_k - \sigma + 1}{\sigma \gamma_k} \right) R_k \quad (15)$$

Finally, consumers' revenues can be expressed as

$$X_k = w_k L_k + \Pi_k = w_k L_k + \frac{\sigma - 1}{\gamma_k \sigma} R_k \quad (16)$$

Using (15) and (16) in (12) yields:

$$R_k = \sum_j \left(\frac{\mathcal{P}_{kj}}{\mathcal{P}_j} \right)^{1-\sigma} \times \left[(1-\alpha) \left(\frac{\sigma\gamma_j - \sigma + 1}{\sigma\gamma_j} \right) R_j + \beta w_j L_j + \beta \frac{\sigma-1}{\gamma_j\sigma} R_j \right]$$

Which can be arranged as:

$$R_k = \sum_j \left(\frac{\mathcal{P}_{kj}}{\mathcal{P}_j} \right)^{1-\sigma} \left[\frac{(1-\alpha)(\sigma\gamma_j - \sigma + 1) + \beta(\sigma-1)}{\sigma\gamma_j} \right] \times R_j + \sum_j \left(\frac{\mathcal{P}_{kj}}{\mathcal{P}_j} \right)^{1-\sigma} \times \beta w_j L_j \quad (17)$$

This can be written more compactly in matrix form as:

$$(\mathcal{I}_N - \mathbb{P} \circ \mathbb{M}) \cdot \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix} = \beta \cdot \mathbb{P} \cdot \begin{pmatrix} w_1 L_1 \\ \vdots \\ w_N L_N \end{pmatrix} \quad (18)$$

Where \circ is the element-wise (Hadamard) product and \mathbb{P} is a matrix defined by $\mathbb{P}_{ij} = \left(\frac{\mathcal{P}_{ij}}{\mathcal{P}_j} \right)^{1-\sigma}$. Moreover, the matrix \mathbb{M} is defined at any time t as:

$$\mathbb{M}_{i,j} = \frac{(1-\alpha)(\sigma\gamma_j - \sigma + 1) + \beta(\sigma-1)}{\sigma\gamma_j} \quad (19)$$

We can then solve for the threshold productivity in closed form to get:

$$\overline{\varphi}_{ki} = \left(\frac{\sigma \frac{c_k}{Z_k} f_{ki}}{\left(\tau_{ki} \frac{\sigma}{\sigma-1} c_k \right)^{1-\sigma} \mathcal{P}_i^{\sigma-1} \cdot [(1-\alpha)S_i + \beta X_i]} \right)^{1/(\sigma-1)} \quad (20)$$

Computational Algorithm

Since the outside good is produced with Constant Returns to Scale and it is freely traded, the price of the outside good is the same in all countries. Hence, the wage is the marginal productivity of labor for production of the outside good: $w_k = Z_k$. The computational algorithm consists in finding a fixed point in threshold productivity types for each directed country pair

$$Phi = \begin{bmatrix} \overline{\varphi}_{11} & \overline{\varphi}_{12} & \dots & \overline{\varphi}_{1N} \\ \overline{\varphi}_{21} & \overline{\varphi}_{22} & \dots & \overline{\varphi}_{2N} \\ \vdots & & & \vdots \\ \overline{\varphi}_{N1} & \overline{\varphi}_{N2} & \dots & \overline{\varphi}_{NN} \end{bmatrix}$$

o. Initial guess of Phi ,

1. Given Φ , solve for a non-zero¹² solution x (which will allow to compute the price indices in the next step) in

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad \text{and} \quad \mathbf{P} = \begin{bmatrix} \lambda_1 \tau_{11}^{1-\sigma} \overline{\varphi}_{11}^{-(\gamma_1+1-\sigma)} & \lambda_2 \tau_{21}^{1-\sigma} \overline{\varphi}_{21}^{-(\gamma_2+1-\sigma)} & \dots & \lambda_N \tau_{N1}^{1-\sigma} \overline{\varphi}_{N1}^{-(\gamma_N+1-\sigma)} \\ \lambda_1 \tau_{12}^{1-\sigma} \overline{\varphi}_{12}^{-(\gamma_1+1-\sigma)} & \lambda_2 \tau_{22}^{1-\sigma} \overline{\varphi}_{22}^{-(\gamma_2+1-\sigma)} & \dots & \lambda_N \tau_{N2}^{1-\sigma} \overline{\varphi}_{N2}^{-(\gamma_N+1-\sigma)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1 \tau_{1N}^{1-\sigma} \overline{\varphi}_{1N}^{-(\gamma_1+1-\sigma)} & \lambda_2 \tau_{2N}^{1-\sigma} \overline{\varphi}_{2N}^{-(\gamma_2+1-\sigma)} & \dots & \lambda_N \tau_{NN}^{1-\sigma} \overline{\varphi}_{NN}^{-(\gamma_N+1-\sigma)} \end{bmatrix}$$

where $\lambda_j = \frac{\gamma_j}{\gamma_j+1-\sigma} M_j \left(\frac{\mu_j}{Z_j}\right)^{1-\sigma}$

2. Compute

- (a) Price index of intermediates $\mathcal{P}_{i,f}$ for $i = 1, \dots, N$

$$\mathcal{P}_i = \frac{1}{x_i^{\frac{1}{\sigma-1}}}$$

- (b) Bilateral price index of intermediates

$$\mathcal{P}_{ki,f}^{1-\sigma_f} = \frac{\gamma_k}{\gamma_k+1-\sigma_f} \mathcal{M}_k \left(\frac{\mu_{k,f}}{Z_k}\right)^{1-\sigma_f} \tau_{ki}^{1-\sigma_f} \underline{\varphi}_k^{\gamma_k} \overline{\varphi}_{ki}^{-(\gamma_k+1-\sigma_f)} \left(\mathcal{P}_{k,f}^{1-\sigma_f}\right)^{1-\alpha}.$$

- (c) Price index of final goods

$$\mathcal{P}_{k,c}^{1-\sigma_c} = \sum_j \frac{\gamma_j}{\gamma_j+1-\sigma_c} M_j \left(\frac{\mu_{j,c}}{Z_j}\right)^{1-\sigma_c} \tau_{ji}^{1-\sigma_c} \underline{\varphi}_j^{\gamma_j} \overline{\varphi}_{jk}^{-(\gamma_j+1-\sigma_c)} \left(\mathcal{P}_{j,f}^{1-\sigma_c}\right)^{1-\alpha}$$

3. Given $\mathcal{P}_{i,f}$ from the previous step, compute the marginal cost c_i ,

$$c_i = \frac{1}{Z_i} \frac{w_i^\alpha \mathcal{P}_{i,f}^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}},$$

and the country level fixed costs:

$$FC_k = \mathcal{M}_k \underline{\varphi}_k^{\gamma_k} c_k \sum_i f_{ki} \overline{\varphi}_{ki}^{-\gamma_k}.$$

4. Solve for country level profits and revenues

¹² $x_i = \frac{1}{p_i^{\sigma-1}}$ so the zero solution is not admissible.

(a) Populate matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} \quad \text{where} \quad \mathbf{A}^1 = \begin{bmatrix} A_{11}^1 & A_{12}^1 & \dots & A_{1N}^1 \\ A_{21}^1 & A_{22}^1 & \dots & A_{2N}^1 \\ \vdots & & & \vdots \\ A_{N1}^1 & A_{N2}^1 & \dots & A_{NN}^1 \end{bmatrix}$$

and, likewise for \mathbf{A}^2 .

$$A_{kj}^1 = \frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j - FC_k .$$

$$A_{kj}^2 = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta w_j L_j .$$

Also,

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}^1 & \mathbf{B}^2 \\ \mathbf{B}^3 & \mathbf{B}^4 \end{bmatrix}$$

where $\mathbf{B}^1, \mathbf{B}^2, \mathbf{B}^3$ and \mathbf{B}^4 are $N \times N$ matrices whose values are

$$B_{kj}^1 = \frac{1}{\sigma_c} \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{kj}^2 = \frac{1}{\sigma_f} \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{ki}^3 = \left(\frac{P_{kj,c}}{P_{j,c}} \right)^{1-\sigma_c} \beta - \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

$$B_{ki}^4 = \left(\frac{P_{kj,f}}{P_{j,f}} \right)^{1-\sigma_f} (1 - \alpha)$$

(b) Solve the system of equations

$$\begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} \mathbf{1} + \begin{bmatrix} \mathbf{B}^1 & \mathbf{B}^2 \\ \mathbf{B}^3 & \mathbf{B}^4 \end{bmatrix} \begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix}$$

so

$$[\mathbf{I} - \mathbf{B}] \begin{bmatrix} \mathbf{\Pi} \\ \mathbf{R} \end{bmatrix} = \mathbf{A} \mathbf{1}$$

where

$$\mathbf{\Pi} = \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_N \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

5. Compute consumers and firms spending (X_k and S_k respectively) using:

$$X_k = w_k L_k + \Pi_k$$

$$S_k = R_k - \Pi_k.$$

6. Compute the threshold type for each bilateral pair in Φ_{ki}' by finding the root φ_{ki} of the following equation

$$\begin{aligned} f_{ki} c_k &= \frac{1}{\sigma_c} \left(\frac{\sigma_c}{\sigma_c - 1} \tau_{ki} c_k \right)^{1-\sigma_c} \left(\frac{1}{\varphi_{ki}} \right)^{1-\sigma_c} \mathcal{P}_{i,c}^{\sigma_c-1} \cdot \beta X_i \\ &+ \frac{1}{\sigma_f} \left(\frac{\sigma_f}{\sigma_f - 1} \tau_{ki} c_k \right)^{1-\sigma_f} \left(\frac{1}{\varphi_{ki}} \right)^{1-\sigma_f} \mathcal{P}_{i,f}^{\sigma_f-1} \cdot (1-\alpha) S_i \end{aligned}$$

Recall that we need to make sure that all thresholds are above 1.

7. If the thresholds Φ_{ki}' are sufficiently close to the initial guess Φ_{ki} , stop. Otherwise, update Φ_{ki}' and repeat steps 1-7.

$$\bar{\varphi}_{ki}^{\text{next iteration}} = \text{smooth} \bar{\varphi}_{ki} + (1 - \text{smooth}) \bar{\varphi}_{ki}'$$

where smooth denotes a smoothing parameter.